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Design and Implementation of an Optimization Model to Improve the Customer Service Level at Minimal Inventory Cost at the EFP GmbH

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Master Thesis

Design and Implementation of an Optimization Model to Improve the Customer Service Level at Minimal Inventory Cost at the EFP GmbH

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Master of Science in International Management

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I. Executive Summary

This paper presents an optimization model designed for the EFP GmbH[1]. It is capable of consistently improving the customer service level (CSL) for the company's biggest customer by ten to twenty percent, without increasing costs. The positive impact on the total CSL is estimated to be about four percent.

The model is based on a non-linear optimization algorithm, which determines at the beginning of the month t+1, for a defined set of critical products, the optimal material quantities to have on stock on the beginning of the month t+3, based on the demand data of the past thirty six months, ending in month t. Thus, the EFP GmbH disposes of sixty days to replenish its stocks accordingly, which is an appropriate period in view of the material lead times.

The model has been implemented in a fully automated system integrated in the company's IT environment. This enables an easy download of input data from the production system MFGPro into the model, as well as a quick upload of the optimized data back into the production system. Operating and maintaining the system takes about one hour per month by one employee.

The model specifically addresses the issue of the so far unplanned short-term order entries, recognized as causing about half of all late deliveries. It does not deal with other causes for back-orders, such as production bottlenecks, supplier capacity constraints, or cross-order material cannibalization. These factors may negatively affect the model results under changing circumstances. Thus, it is important to take action in respect of those issues.

Based on the results of a twelve month test period, the model appears to be valid. As changes in the environment occur, it remains important to monitor the model validity on an annual basis. In the future, the model may also be extended to drop-in products of other customers, so as to increase the impact on the total customer service level.

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[1] The real name of the project client has been replaced for confidentiality.

II. Acknowledgments

I would like to thank my academic project advisor Prof. Dr. Rainer Kämpf, Professor of Operations Management at the European School of Business in Reutlingen (Germany). He gave me precious advice with regard to the literature and research methodology, and supported me in the elaboration of the Thesis' structure and design. My professional project advisor, Stefan Erdmann, Plant Manager of the EFP GmbH (Germany), provided me with the information, but also with the material and financial means necessary to carry out this project in only three months. His enthusiasm and interest in my work have always been a great source of motivation. I also thank Bart Merkx, EFP Lean and Six Sigma (ELSS) Manager at the EFP GmbH, who has been a great colleague to work with and who was always there to support and motivate me when needed. I want to thank Mark Wiley, with Lindo Systems Inc., for providing me with a free license of the optimization software What'sBest!®. Although I ultimately could not use this solver in this project, he was always ready to help and to work out solutions together. I am very grateful for the help of my roommate and friend Tobias Waldschmidt for his precious suggestions in matters related to statistics and programming. Finally, I would like to thank all the employees of the EFP GmbH who helped me bring together the information necessary for this project.

Many thanks to all!

Stefan Gebauer

III. Table of Contents

IV. Chart Index

V. Equation Index

VI. Abbreviation Index

B2B	Business to Business
CI	Confidence Interval
CSL	Customer Service Level
DM	Data Module
DOH	Days on Hand
EFP	Designation for the project client
ELSS	EFP Lean & Six Sigma
ES	Exponential Smoothing
FM	Forecasting Module
ICC	Inventory Capital Costs
LBH	Designation for EFP’s biggest customer
MA	Moving Average
MAD	Mean Absolute Deviation
ME	Mean Error of the Forecast
MLT	Material Lead Time
NM	Naïve Method
OM	Optimization Module
OR	Operations Research
OT	Order Time
OTD	On Time Delivery
PLT	Product Lead Time
SE	Standard Error of the Forecast
ST	Shipping Time
SV	Storage Volume
VBA	Visual Basic for Applications

1. Introduction

1.1. Project Topic and Aim

The vast majority of manufacturing companies suffer from lower customer service and higher inventory costs than are necessary. In this paper, customer service is defined as the on-time delivery of the right quantity of products of the right quality[2]. The major causes for this are the use of poor inventory control processes and antiquated support systems (Donovan, 2007). This fact is exacerbated when demand is strongly volatile and consequently difficult to predict.

The EFP GmbH derives the largest part of its sales from make-to-order hose assemblies and pipelines for both the agricultural and construction industries. In accordance with the high degree of customization and the small lot sizes of its business, the firm operates in a batch production system, in which many different jobs are performed at the same time in groups (Russell/Taylor, 2000). While many of the company's customers provide medium term forecasts and place early orders, EFP's biggest customer (henceforth LBH[3]) drops in about 31%[4] of its orders only few days before the requested delivery date. So far, no system is in place to analyze and forecast these short-term order entries in order to timely order the material, hence regular inventory stock-outs and frequent back-orders.

As for many companies, the continuous improvement of customer service is a central objective of the EFP Corporation, to which belongs the EFP GmbH. On the other hand, the reduction of fixed and working capital in order to increase operative margins is a major aspect of the corporate strategy.

The aim of this project is to design and implement a model that provides an effective but also efficient approach to the typical conflict of aims between inventory costs and customer service (Mohr, 1993). In accordance with the company's strategy and needs, and taking into consideration the data availability, this paper introduces a non-linear optimization model (henceforth referred to as "the model") that statistically assures a predefined level of customer service for drop-in parts[5] ordered by LBH while keeping inventory costs down to a minimum.

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[2] See also Definition 2.1.3 in Chapter 2.

[3] The real customer name has been replaced for confidentiality.

[4] From 01/01/05 to 31/12/06, 30.98% of LBH orders came in 10 days or less before due.

[5] i.e. short-term order entries, see also Definition 2.1.5 in Chapter 2.

1.2. Literature and Methodology

1.2.1. Preliminary Literature Research

The preliminary literature research helped to identify a set of tools that would support the EFP Management to achieve its optimization target. To begin with, fundamental literature about operations management, business statistics, and business forecasting has been screened. Traditional concepts and methods, such as statistical tests and forecasting methods that will be integrated into the model, are described in detail in Chapter 2.

A more recent field of study, Operations Research (OR), is concerned with identifying the optimal action alternative in generally very complex (business) situations (Werners, 2006). The task being here to *optimize* one variable (i.e. to minimize inventory costs) under the constraint of another (i.e. to guarantee a given level of customer service), optimization algorithms were identified as promising tools with regard to the project target (Kathöfer / Müller-Funk, 2005). Relevant OR methods and concepts are presented in greater detail in Chapter 2.

It must be highlighted that the model presented in this study uses existing statistical concepts and OR techniques, but is completely new in the way of combining them in order to achieve the EFP GmbH's goals.

1.2.2. Practical Application of Theoretical Concepts: Methodology

1.2.2.1. Data Gathering and Model Designing

The gathering of real data within the company required a thorough analysis of the information systems as well as interviews with system experts. EFP is working with non-integrated information systems (as briefly described in Chapter 3), so that some data have not been recorded historically or are not available in the form needed for the preliminary model concept.

In agreement with Management, the initial model design was thus reviewed to account for missing data while still satisfying the company's needs. In particular, the model was simplified with regard to the number of constraints. Also, decisions were made about how to select the products to be included in the model, which methods to use for the computation of the demand forecast, and how often to update the forecast data. All model parameters and the reasons for their selection are presented and explained in detail in Chapter 4.

1.2.2.2. Model Implementation

After redesigning the model in accordance with the company goals but also the system and data environment, the model was implemented with the objective to be simple to understand and practical (easy and quick) to use on a monthly basis. This required the programming of a system integrated in the current IT environment of the firm. In particular, the data gathering from the existing systems, the data processing in new MS Access databases, the model solving via MS Excel (see next section), and the output data upload back into the existing system were automated using the VBA programming language. A more detailed description of the model implementation is given in Chapter 5.

1.2.2.3. Optimization Algorithm for Model Solving

Unfortunately, existing optimization applications for MS Excel, such as "*What's Best® 8.0*", which provide both linear and non-linear solver algorithms, do not support the probability distribution (see 2.2.2) functions needed for the model. Therefore, an optimization algorithm has first been programmed in VBA that is suited for solving the model as previously designed. The functioning and validity of this algorithm is mathematically demonstrated in Appendices 1 to 4.

1.2.2.4. Model Evaluation and On-Going Assessment

The last phase of the project is the model evaluation and the implementation of a system for an on-going monitoring of the model validity. As presented in Chapter 6, the evaluation is based primarily on a statistical hypothesis test, assessing the hypothesis that the average CSL achieved using the model is equal to the defined target CSL (sample test period is from July 1st 2006 to June 30th 2007). Additionally, a historical comparison is made over this test period between the CSL for LBH drop-in products that was effectively achieved and the CSL that would have been achieved using the model. This comparison is complemented by an estimation of the inventory costs necessary to achieve this customer service improvement.

1.3. Expected Project Result

As defined by EFP Management, the output of this project is to be an information system based on an optimization model integrated into the company's existing IT environment. The system is expected to be capable of *easily* and *timely* deliver information about the cost-optimal quantity of material needed on stock on the first day of each month in order to achieve the wanted level of customer service for LBH drop-in products.

1.4. Paper Overview

Chapter 2: Theoretical Background

This chapter defines the terms of the thesis title which are used throughout the paper and relevant to understand the purpose of the model. It also explains the statistical concepts later used in the model construction. Business forecasting techniques and criteria for choosing an appropriate technique are presented, too. Moreover, the concepts of linear and non-linear optimization on which the model is based are explicated.

Chapter 3: EFP GmbH – Objectives, Results and Issues

This chapter provides the information about the business environment at EFP relevant for the development of the model. First of all, Management goals with regard to customer service and inventory costs are defined and compared to the actual results. Then, current issues in operations and inventory management that hinder the reaching of the objectives are pointed out and analyzed. It is then stated which of the goals the model pursues and which issues it addresses. Also, the IT systems at EFP are described for a better understanding of the system environment.

Chapter 4: Model Design, Parameters and Assumptions

This chapter presents the model design, from the conceptual creation down to the dataflow analysis, always referring to the theoretical concepts exposed in Chapter 2, and taking into account the specific management needs and data constraints. Also, all model parameters and assumptions that were made are explained and justified.

Chapter 5: Model Implementation

In this chapter, the implementation process of the model is described. Especially, the system structure, interfaces, functionalities, maintenance, but also limitations are depicted.

Chapter 6: Model Evaluation and Validity Monitoring

In this chapter, the design and results of the statistical hypothesis test, as well as the historical comparison and cost estimation previously mentioned in the methodology section of this paper (see 1.2.2.4), are presented.

Chapter 7: Conclusion and Critical Outlook

This chapter concludes the report by critically evaluating the performance and results to be expected from the designed and implemented model. It also critically considers the system limitations and their potential impact on the model's performance.

2. Theoretical Background

2.1. Important Definitions

The following are concepts used in or related to the title of this paper. Their definition shall ease the understanding of the purpose and content of this paper.

2.1.1. Model / Modeling

A model is a simplified representation of generally more complex real-world relationships (Kluck, 1998) with the purpose of either explaining reality, predicting reality, or supporting real-life decision-making (Derigs/Ems, 1999). Models can take the shape of charts, drawings, prototypes, but also mathematical models like regression or optimization models. In this paper, the model to be developed is a mathematical optimization model aiming at helping the EFP GmbH in making decisions regarding their inventory and customer service management.

Kathöfer & Müller-Funk (2005) describe *modeling* as a three-stage procedure starting with the formulation of the problem, followed by the process execution, and ending with the validation of the results. The formulation of the problem is generally done in words at first, then translated into a mathematical model with variables, constraints, and relationships (Chapter 4). The process execution is the actual computation phase, where data are entered in the model to obtain an output (Chapter 5). Finally, it must be assessed whether the results are meaningful and useful with regard to the problem at hand (Chapter 6).

2.1.2. Optimization / Optimum

Optimization is the process of identifying the feasible[6] action alternative(s) that correspond(s) best to the defined goal (Werners, 2006). Whether there is one or more such solutions depends on the model at hand. In particular, one must distinguish between global and local optima (Vrhovec, 2001). In linear optimization models, there is only one globally optimal solution situated at the boarder of the feasible solution area (Kathöfer/Müller-Funk, 2005). When non-linearity is present, an algorithm may only be able to find a locally optimal solution. A local optimum is a point from which a small change in variables in any dimension leads to a worse result (Vrhovec, 2001). However, there may then be other local optima within the feasible solution area. Although the present paper presents a

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[6] In mathematics, a feasible solution is one that satisfies all given constraints.

non-linear optimization model, it will be demonstrated that the employed algorithm leads to the global optimum, with a definable level of accuracy.

2.1.3. Minimal Inventory Cost

In this paper, optimization actually means *minimization* of inventory costs under given constraints. The actual cost of holding stock is composed of different types of costs, mainly including tied-up capital costs, insurance and rent, obsolescence and handling/auditing costs (Barnett, 1996). As the present model is focusing on a small share of critical products, it can be assumed that all but the capital costs can be considered as fix and thus not relevant for the optimization.

For the products of the model, material is ordered so as to be on stock on the first day of each month, with no more material arriving during the month. Be I_1 the inventory value on the first day, I_s the portion of inventory that is sold during the month, and I_2 the remaining inventory value on the last day of the month (i.e. $I_2 = I_1 - I_s$). Assuming for simplicity a constant usage of the stock, and given a monthly capital return of r%, the monthly inventory capital costs (ICC) can be computed as follows:

$$\textbf{(1.)}\qquad ICC = r\cdot\left(\frac{I_1 - I_2}{2} + I_2\right) = r\cdot\left(\frac{I_1 - (I_1 - I_s)}{2} + I_1 - I_s\right) = r\cdot\left(I_1 - \frac{I_s}{2}\right)$$

Given that the quantity of inventory that is really sold (I_s) depends on demand, a factor assumed out of EFP's control, it appears from equation 1 that minimizing inventory costs (ICC) is achieved by optimizing (i.e. minimizing) the cost of materials that need to be on stock on the first day of each month (I_1). Under the constraint of achieving a certain level of customer service (see next section), this is precisely what the model has been designed to do.

2.1.4. Customer Service Level (CSL) / On-Time Delivery (OTD)

2.1.4.1. EFP Reporting Definition

Broadly speaking, customer service comprehends all services provided to customers before, during and after a purchase (Kluck, 1998). In this paper however, as often when speaking about customer service in the context of inventory management (Donovan, 2007), the meaning has been narrowed down to the on-time delivery of the products in the number and quality requested by the client. In EFP reporting, the CSL (or OTD) is computed as follows:

(2.) $$CSL = \frac{(TO - BO)}{TO}, \quad \text{where} \left| \begin{array}{l} TO\text{: Total number of orders} \\ BO\text{: Number of backorders (late delivery)} \end{array} \right.$$

An order is defined as a single position on the actual order sheet. If a customer orders three products A, B, and C at on time (i.e. on one single order sheet with one single order number), these are considered to be three orders as regards the computation of the on-time delivery. This means, if e.g. A and B are delivered on time but C is not, then $CSL = (3 - 1) / 3 = 66.66\%$. This is due to the possibility of partly delivering finished order positions to customers.

2.1.4.2. Model Adapted CSL Definition

According to the CSL reporting definition, it is not relevant how many orders for one same product are placed in the same month (e.g. if two orders were made for product A, both due in January, and one of them was delivered on-time, then the CSL would be 50%). However, the model determines the optimal quantity corresponding to the forecasted demand for a complete month, but cannot possibly foresee on how many orders this demand may be split. Thus, the CSL for a given product is calculated by the model as if the total monthly demand was one single order. Considering the example above, the model would thus compute a CSL of 0% for product A in January, as a part of the monthly demand was delivered late.

It will be shown in the model evaluation (see 6.1.2.1) that, over time, the differences in CSL caused by the application of the one or the other of the two definitions offset each other and are thus negligible.

2.1.5. Order Time / Drop-In Orders

In this paper, the order time is defined as the duration in working days between the order date and the due date. Inventory management becomes crucial when the order time does not allow a company to order materials from its suppliers upon order entry, produce the order, and ship it timely to its customer. In other words, materials must be kept in stock as soon as:

(3.) $$\text{Order Time} < \text{Material Lead Time} + \text{Production Lead Time} + \text{Shipping Time}$$

A drop-in order in the sense of this report is an order that satisfies equation 3. If enough material is on stock on the day of the customer order entry, the order will be timely delivered whenever:

(4.) Order Time > Production Lead Time + Shipping Time

The assumptions used for order and lead time calculations in the framework of the model are presented in the model design (see Chapter 4).

2.2. Statistical Concepts and Tools

The following fundamental statistical tools will be used or referred to later in the model design, implementation and evaluation.

2.2.1. Descriptive Statistics

Descriptive statistics are a few key summary values used to describe data collections (Hanke / Reitsch, 1995). The basic concepts that will be used later in this paper are defined here.

a) Population (μ) and Sample ($\bar{Y}$) Mean

The mean is computed by adding all values of a data series and dividing the sum by the number of values (N if the population is considered, n for a sample):

(5a.) $$\mu = \frac{1}{N} \cdot \sum_{i=1}^{N} Y_i$$

(5b.) $$\bar{Y} = \frac{1}{n} \cdot \sum_{i=1}^{n} Y_i$$

b) The Median ($\tilde{Y}$)

The median is the value dividing a data series so as to have an equal number of values superior and inferior to that value. If there is an even number of values in the series, the mean of the two central values is taken as the median. It is a valuable measure when dealing with skewed data series, as it is not influenced by outliers (Chart A).

Chart A: Mean vs. Median

Y	36	40	42	44	44	45	64	85

Mean: $\bar{Y} = (36 + 40 + 42 + 44 + 44 + 45 + 64 + 85) / 8 = 50$

Median: $\tilde{Y} = (44 + 44) / 2 = 44$

The mean is significantly higher than the median because, unlike the median, it takes into account all data points, including the outlier 85.

c) Population (σ) and Sample (s) Standard Deviation

The standard deviation is used to measure the extent to which the values are dispersed around the mean (Black / Eldredge, 2001). The notation and formula differ depending on whether a whole population or only a sample is considered (Hanke / Reitsch, 1995).

(6a.) $$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (Y_i - \mu)^2}{N}}$$ **(6b.)** $$s = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}{n-1}}$$

2.2.2. Probability Distributions

Fundamentally, one must distinguish between discrete and continuous probability distributions, in which a random variable can respectively assume either only certain specified values (e.g. integers between 1 and 10) or any numerical value within some range (Anderson et al., 2004).

In this paper, both types of probability distributions are used: the Binomial distribution (discrete), the Normal distribution (continuous), and Student's *t* distribution (continuous).

a) Binomial Distribution

A binomial experiment can be done if and only if the three following conditions are satisfied: 1) the experiment consists of n identical trials, each of which has the two same possible outcomes; 2) the probability of each outcome remains fix for each trial; and 3) the trials are independent. The aim of the experiment is to determine the probability of having x times one of the two outcomes, called *success*, when doing n trials (Hanke / Reitsch, 1995).

This probability can be computed using equation 7.

(7.) $$P(x) = C_x^n \pi^x (1-\pi)^{n-x}$$

for $x = 0, 1, 2, \ldots, n$, where:
x is the number of successes
π is the probability of success
n is the number of trials

Alternatively, a binomial table can be used where these probabilities can directly be read given n, π, and x (see example in Appendix 5).

b) Normal Distribution

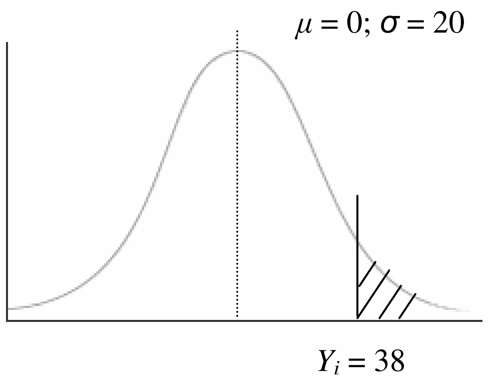
Because many populations can be approximated by it, the most used continuous distribution is the normal distribution, represented by a Gauss curve (Chart B), and characterized by a mean (μ or $\bar{Y}$) and a standard deviation (σ or s) (Hanke / Reitsch, 1995). The calculation of probabilities of normally distributed values Y_i $(i = 1, 2, \ldots, n)$ requires their prior conversion to so-called z-scores. The

z-score of any Y_i value corresponds to the number of standard deviations between the mean and that value (Black / Eldredge, 2001). It is computed as follows, for a population and a sample value respectively:

(8a.) $z = \frac{Y_i - \mu}{\sigma}$ **(8b.)** $z = \frac{Y_i - \overline{Y}}{s}$

A z-table or MS Excel can then be used to compute different probabilities around that value, depending on the question to investigate. An example is given in Chart B.

Chart B: The Normal Distribution



What is the probability of obtaining a value greater than 38 out of this population of $N = 1000$?

$z = \frac{38 - 0}{20} = 1.9$ (see equation 8a)

In the z-table, a z-score of 1.9 corresponds to a probability of 2.87% of obtaining a higher value (dashed area).

c) Student's t-Distribution

However, the so-computed probabilities are only accurate for large populations or samples. William Gosset (alias Student) demonstrated that, as N (n) becomes small, the distribution remains bell-shaped but becomes flatter (Dallal, 1999). This means that as N (n) decreases, the probability of obtaining a value further away from the mean increases. Although there is only one normal distribution, there are thus an infinity of so-called t-distributions, that vary in function of the size of the population or sample at hand (Chart C). More precisely, they depend on the degrees of freedom (df), defined as:

(9a.) $df = N - 1$ **(9b.)** $df = n - 1$

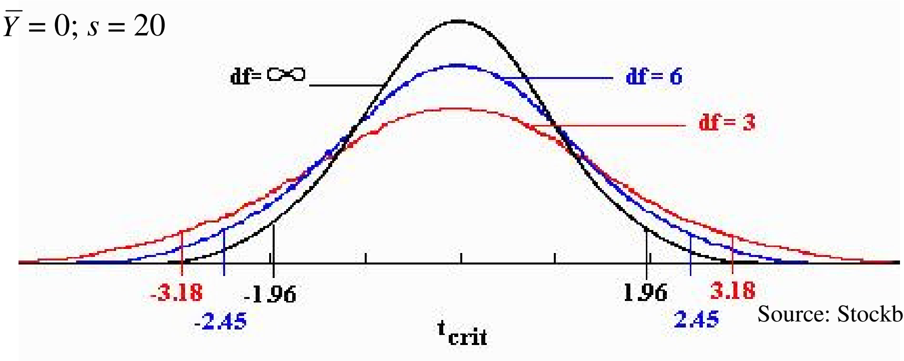
As a general rule, t-distributions can be approximated by the normal distribution as soon as $df \geq 120$, although they remain valid even for very large population or sample sizes. Again, the first step for computing probabilities is to convert the Y_i values into t-values as follows:

(10a.) $t = \frac{Y_i - \mu}{\sigma}$ **(10b.)** $t = \frac{Y_i - \overline{Y}}{s}$

Then, using MS Excel or a t-table, the associated probability can be computed in function of the degrees of freedom, as the example in Chart C illustrates.
In the model presented in this paper, observed samples are always small. For that reason, the t-distribution is applied rather than the normal distribution.

Chart C: Student's t-Distributions (examples for *df*=3 and *df*=6)

$\bar{Y} = 0$; $s = 20$



Source: Stockburger

Is the probability of obtaining a value outside the range [-44 ; 44] higher or lower than 5% for a sample of n=1000? n =6? n =3?

The t-value range is, with $t = \frac{44-0}{20} = 2.2$, equal to [-2.2 ; 2.2].
The critical t-values (i.e. t-values forming a 95% confidence interval around the mean), have been written on the graph for each curve.

Compared to t_{crit}: 2.2 > 1.96 but 2.2 < 2.45 and 2.2 < 3.18.

Consequently, the probability of obtaining a value outside the given range is lower than 5% for n=1000, but higher than 5% for n=6 and n=3.

2.2.3. Sampling Distributions

A sampling distribution is the distribution of all values of a sample statistic (e.g. the mean) that can be obtained by taking samples of a given size from a population (Hanke / Reitsch, 1995). According to the Central Limit Theorem, the sampling distribution of sample means tends toward a normal distribution, qualified by a mean equal to the population mean and a standard deviation called the standard error of the sampling distribution and defined as (Anderson et al., 2004):

(11a.) $\mu_{\bar{Y}} = \mu$ **(11b.)** $\sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{n}}$ (if σ is unknown: $\sigma_{\bar{Y}} = \frac{s}{\sqrt{n}}$)

A sampling distribution is used whenever conclusions are to be drawn about a population statistic (e.g. μ) based on a sample statistic (e.g. $\bar{Y}$), generally using confidence intervals or hypothesis tests. If the population standard deviation is not known, the sample standard deviation can be used as a point estimate. If $n \leq 120$, the t-distribution should be used instead of the normal distribution for approximating the sampling distribution (Dallal, 1999).

2.2.4. Confidence Intervals

A confidence interval (CI) is an interval of likely estimates for a population parameter. It is most often used to estimate the population mean based on a sample mean, which constitutes one point value of the normally distributed sampling distribution of the sample means. The computation of the CI is shown in equations 12a and 12b, the latter being used whenever the population standard deviation is unknown and/or $n \leq 120$ (see 2.2.2).

(12a.) CI: $\mu \pm z \cdot \sigma_{\bar{Y}}$ **(12b.)** CI: $\mu \pm t \cdot \sigma_{\bar{Y}}$

CI are also applied in other contexts. In the model presented later, a CI is built around the forecast ($\hat{Y}_t$) in function of the forecasting standard error (equal to the standard deviation of the forecast errors (SE)), in order to know what needs to be on stock for having a probability α % of stocking out. As the population standard deviation is unknown and only twelve forecasting errors are considered (hence $df = 12 - 1 = 11$), this CI is computed by $\hat{Y}_t \pm t_{(\alpha = \alpha\%,\, df = 11)} \cdot SE$.

The difference between the upper boarder of the CI and $\hat{Y}_t$ is then nothing else but the security stock (Vollman et al., 1997).

2.2.5. Hypothesis Testing

Hypothesis testing is used to verify some claim (for example, the mean equals zero) about a population. The test involves five successive steps (Chart D):

Chart D: The Five Steps of Hypothesis Testing

1. Define the null hypothesis (H_0) and the alternative hypothesis (H_1).
2. Define a level of significance (α) and compute the corresponding critical statistic (e.g. t_{crit})
3. Formulate a decision rule (i.e. define for what values of the test statistic H_0 would be rejected).
4. Compute the test statistic (e.g. t-value) from a random sample of the population.
5. Make the decision about H_0.

Source: Triola (2004)

The critical and the test statistics are t-values in the present paper, as the model is dealing with small samples and population means are not known. The model evaluation in Chapter 6 (see 6.1.1) constitutes an applied example of a hypothesis test assessing whether the average CSL achieved with the model is equal to the predefined target CSL.

2.3. Business Forecasting

In this section, the notion of business forecasting is introduced along with the forecasting techniques used later in the model. Also, the criteria used to assess the adequacy and accuracy of such techniques are presented.

2.3.1. The Forecasting Process

Hanke & Reitsch (1995) describe forecasting as a four-step process: data collection is followed by data reduction and condensation, then the forecasting model is built and finally the model is extrapolated, i.e. forecasts are made. The first two steps highlight the importance of having access to the relevant data and to select the most meaningful data with respect to the forecasting objective (see 2.3.2). The forecasting model building involves the testing of different forecasting techniques and the selection of the most appropriate one(s) based on various factors (see 2.3.3). The last step refers to the actual using and on-going assessment of the created model. It is worth underlining that the forecasting model is only a sub-model of the optimization model presented in this report.

2.3.2. About Forecasting Data

For obvious reasons, the quality of a forecasting model greatly depends on the quality of the data input. The major characteristics of useful input data are *accuracy* (the data are correct), *relevance* (the data are representative of the real situation), *consistence* (the data are always collected from the same source and computed in the same fashion) and *timeliness* (the data are available periodically when needed for forecasting purposes) (Hanke / Reitsch, 1995).

2.3.3. Forecasting Techniques

2.3.3.1. Overview of Forecasting Techniques

Fundamentally, one must distinguish between *qualitative* (mainly relying on human judgment) and *quantitative* (mainly relying on statistical tools) forecasting techniques (Russell / Taylor, 2000). Quantitative methods require both the availability of enough historical data and the assumption that those are representative of the unknown future (Black / Eldredge, 2001). In the present model, both conditions are satisfied, and only quantitative forecasting techniques are used in accordance with the mathematical nature of the optimization model (see 2.1.1).

Quantitative techniques are frequently classified into statistical and deterministic models. Whereas statistical techniques (e.g. smoothing techniques) focus on patterns and pattern changes caused by random influences, deterministic techniques (e.g. regression analyses) suppose relationships between the (dependent) variable to be forecasted and other (independent) variables (Madura, 2002). During preliminary research, no causal relationships could be identified between the demand of individual products or groups of products at the EFP GmbH and external factors. For this reason, the focus of this paper lies on *statistical* (also called probabilistic) forecasting techniques.

When talking about time series forecasting, two other important criteria of differentiation for forecasting techniques are to be mentioned: the forecasting period and horizon (Vollman et al., 1997). The forecasting period is the period *for* which the forecast is made (e.g. week, month, year). Indeed, whereas seasonality may play an important role in monthly or quarterly forecasting, it is not relevant for yearly forecasts. Conversely, cyclical and trend-patterns affect yearly forecasts much more than monthly or weekly forecasts (Billah et al., 2005). The forecasting horizon is the period *over* which the forecast is made in advance (e.g. over the next four weeks, twelve months, ten years). Madridakis (1982) and Armstrong (1984) demonstrated that simple techniques (e.g. smoothing methods) usually outperform more complex methods for short-term forecasting.

2.3.3.2. Short-Term Statistical Forecasting of Time Series

The present optimization model requires the forecasting of the monthly demand three months[7] in advance. To do so, seven common statistical forecasting techniques for short-term time series forecasting have been tested: classical decomposition for seasonality and trend, simple exponential smoothing, trend-adjusted exponential smoothing, four-month moving average, four-month double moving average, trend- and seasonality-adjusted naïve method, and naïve method. The test design and results are presented under 4.3.

The three methods that have been retained for the model based on this assessment are quickly presented here. For each of them, both the standard formula (for one period in advance) and the adapted formula used in the model (for three periods in advance) are indicated. Sample computations for all tested methods and the test results are shown in Appendices 6 and 7, respectively.

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[7] The reason for this forecasting horizon is explained under 4.1.3. This horizon leaves EFP sixty days to replenish its material inventory, which is the maximal lead time for materials.

a) Simple Exponential Smoothing (ES)

Exponential smoothing consists in continuously revising the prior forecast in the light of more recent experience (Hanke / Reitsch, 1995). The extent to which changes in demand pattern are taken into consideration in a new forecast is expressed by the smoothing factor α (Kluck, 1998):

(13.) $\hat{Y}_{t+1} = \alpha Y_t + (1-\alpha)\hat{Y}_t$ [adapted in the model to: $\hat{Y}_{t+3} = \alpha Y_t + (1-\alpha)\hat{Y}_t$]

This factor can be estimated by an iterative procedure that consists of minimizing the mean absolute deviation (MAD, see 2.3.3.3) for past demand data. This can be done using a non-linear optimization model (see Appendix 8).

b) Moving Average (MA)

The moving average method proceeds by averaging a selected number of past periods data (Vollman et al., 1997). The more past periods are considered, the smoother the forecast becomes, i.e. the less reactive it will be to changes in recent demand pattern (Russell / Taylor, 2000).

The moving average forecast computed in period t for the period $t+1$ is, h being the number of historical periods considered ($h = 4$ in the model):

(14.) $\hat{Y}_{t+1} = \dfrac{\sum_{i=t-h+1}^{t} Y_i}{h}$ [adapted in the model to: $\hat{Y}_{t+3} = \dfrac{Y_{t-1} + Y_t + \hat{Y}_{t+1} + \hat{Y}_{t+2}}{4}$]

c) Naïve Method (NM) – Random Walk

Random walk is the simplest forecasting method, as it assumes that the most recent past period is the best predicator for the period to come:

(15.) $\hat{Y}_{t+1} = Y_t$ [adapted in the model to: $\hat{Y}_{t+3} = Y_t$]

This method is accurate when successive values (respectively values lagged three periods as for the model adapted equation) are highly autocorrelated (Hanke / Reitsch, 1995). The autocorrelation factor (R_k) for values lagged k periods is calculated by the following equation (see example in Appendix 9):

(16.) $R_k = \dfrac{\sum_{i=(t-n+1)+k}^{t} (Y_i - \bar{Y})(Y_{i-k} - \bar{Y})}{\sum_{i=t-n+1}^{t} (Y_i - \bar{Y})^2}$

with $t \geq n > k \geq 1$, and $0 \leq R_k \leq 1$
where k is the period lag, t is the last period of the observation, n the number of periods observed, and t-n+1 the first period of the observation

2.3.3.3. *Accuracy and Honesty of a Forecasting Technique*

A very common approach to selecting the technique appropriate to a particular time series is the prediction validation on a withheld part of the data series using criteria such as the mean absolute error (MAD) (Billah et al., 2005). The MAD is a common measure of forecasting accuracy computed as follows:

(17.) $$MAD = \frac{\sum_{t=1}^{n} \left| Y_t - \hat{Y}_t \right|}{n}$$, where Y_t is the actual value and $\hat{Y}_t$ the forecast value

The MAD is an *absolute* error measure, as it represents the average deviation between the forecast and the actual value. There also exist percentage error measures, such as the mean absolute percentage error ($MAPE$). However, percentage error measures can only be used under the condition of having no zero value in the observed times series (Hanke / Reitsch, 1995). Due to frequent zero-demand months, this condition is not satisfied for the time series to be observed in the present study, so that accuracy is to be evaluated by absolute measures.

Accuracy is not the only important measure to evaluate a forecasting technique. For being valid, a forecasting technique must also produce honest, i.e. unbiased forecasts (see example in Appendix 10). This means that the forecasts should be neither consistently high nor low, so that over-forecasts will offset under-forecasts over time (Vollman et al., 1997). By definition, the mean forecasting error (ME) produced by a completely unbiased forecasting technique is to be 0 over time (i.e. for the whole population of errors e_t, for t=1 to N, where N is very large). Additionally, assuming the normal distribution of forecasting errors, the expected number of overestimates is ultimately equal to the expected number of underestimates for a large number of observations.

For assessing the honesty of forecasting techniques in the present paper, two tests have been designed. The first is based on a confidence interval (see 2.2.4) around the mean forecasting error of the population equal to zero. The second is a binomial test based on the computation of the likelihood of the number of positive forecasting errors over the past twelve observations. Both tests are presented in detail in Appendix 5.

(18.) $$ME = \frac{\sum_{t=1}^{n} e_t}{n}$$

Additionally, if the forecasting errors (e_t) are normally distributed[8] around a mean error of 0, the standard deviation of the forecasting errors (i.e. the standard error of the forecast, noted SE) is arithmetically related to the MAD by equation 19b (Vollman et al., 1997, see Appendix 15):

(19a.) $e_t = Y_t - \hat{Y}_t$ **(19b.)** If $e_t \sim \mathrm{N}$ and $ME = 0$, $SE = 1.25 \cdot MAD$

2.3.3.4. Focus Forecasting

In accordance with the observation that simple models work best for short-term forecasting (Madridakis, 1982 & Armstrong, 1984) and with the assumption that the forecasting method that worked best last time may work best this time, the focus forecasting technique consists of always using the one forecasting method that would have performed best in recent periods (Vollman et al., 1997). Practice has shown that focus forecasting is a good means for maintaining steadily accurate forecasts over long periods of time.

2.4. Operations Research: Solving Optimization Problems

Operations Research (OR) is a relatively recent[9] and still very dynamic field of research which aims at the development and practical implementation of quantitative models and methods in order to support decision-making (Werners, 2006). It has grown to be a very broad and hardly precisely definable area of study, with new algorithms and models emerging merely every day to solve more or less reality focused optimization problems (for more information, please consult the OR Homepage listed in the literature review). Focusing on the needs of the current paper, the following section deals only with those concepts relevant for understanding the optimization model presented in the upcoming chapters.

2.4.1. Optimization Problem – Mathematical Definition

An optimization problem is mathematically defined by the following elements (Kathöfer / Müller-Funk, 2005):

- a quantity Z of feasible solutions (i.e. action alternatives), defined by the problem constraints; and

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[8] This assumption will be made in this model based on the observation of a sample of histograms of the forecasting errors produced by the techniques used (Appendix 14).

[9] The concept was first used in the middle of the 20th century, but many of the current tools and methods were developed much later (Werners, 2006).

- a real function $f(X)$, called target function, with $X \in Z$, which represents the factor (e.g. costs, profits) to optimize.

An optimization problem can have two objectives: the maximization (e.g. of profits) or the minimization (e.g. of costs) of the target function (Hüftle, 2006). Taking the example of the present paper, where the objective is cost *minimization*, the optimization problem is formally described by (Werners, 2006):

(20.) $$f(X) \overset{!}{=} \min_{X \in Z} \qquad \text{where } X^* \in Z \text{ is optimal if: } \forall X \in Z,\ f(X^*) \leq f(X)$$

There exist several cases in which no optimal solution can be found. Indeed, an optimization problem cannot be solved (Kathöfer / Müller-Funk, 2005) if:

- Z is empty;
- $f(X)$ is not limited by at least one constraint in the direction of the optimization (i.e., $\forall X \in Z$, there exists an X' such as $f(X') \leq f(X)$); or
- $f(X)$ is limited by at least one constraint in the direction of the optimization, but there does not exist any value of X satisfying this constraint.

Optimization problems can be solved by the means of algorithms, which are introduced in the next section.

2.4.2. Optimization Algorithms

An algorithm is a processing instruction for solving a problem. Nowadays, these instructions generally take the very detailed form of a program (Werners, 2006). There exist three major types of optimization algorithms:

- *effective* algorithms, which lead to the global optimum after a finite number of steps, but may necessitate very long computation times as the model complexity increases;
- *heuristic* algorithms, which lead usually very quickly to an approximate best solution by simplifying the model structure, but there is no guarantee for that solution to be the global optimum;
- *simulation* algorithms, which are at least partly based on random search and also lead to an approximate best solution, which practice has shown to be often very close to the global optimum (Kathöfer / Müller-Funk, 2005).

Which kind of algorithm is best suited for a given optimization problem depends on both the complexity of this problem and the requirements one has regarding the optimality of the solution. In very complex business situations, the efficiency of heuristic and simulation algorithms may be preferable to the accuracy of effective algorithms, as the costs of sub-optimality are offset by the time savings. However, if an effective algorithm can be efficiently applied, it is clearly to be favored over its less precise counterparts.

In the present study, an effective algorithm has been programmed that determines a globally optimal solution at some definable level of accuracy.

2.4.3. Different Optimization Model Types

Depending on the optimization problem at hand, on must distinguish between different types of models. The model type has an influence on the choice of an appropriate algorithm, and consequently also on the nature of the optimal solution (i.e. global vs. local optimum, see 2.1.2).

The most common and central model type in OR is the linear optimization. An optimization problem is linear when both the target and the constraint functions are linear with regard to the model variables (Werners, 2006), i.e. when each variable $X_i \in R^+$, $i \in [1 ; p]$, $p \in N$ is related to the function by $f(X_i) = a_i \cdot X_i$, $a_i \in R^+$. If the target or at least one constraint function are not linear, than the optimization problem forms a non-linear optimization model. Examples of non-linear relationships are given in Chart E. These two fundamental model types can again be constructed in several variants. For instance, one may want to include negative value of X_i into the feasible solution area, i.e. $X_i \in R$, $i \in [1 ; p]$, $p \in N$; one may want to restrict the feasible solution area to positive integers, i.e. $X_i \in N$, $i \in [1 ; p]$, $p \in N$ (integral optimization, (Hüftle, 2006)); or one may have a problem that does only allow a defined set of values $D = \{d_1, d_2, \ldots, d_q\}$, $q \in N$, i.e. $X_i \in D$, $i \in [1 ; p]$, $p \in N$ (discrete optimization, (Kathöfer / Müller-Funk, 2005)).

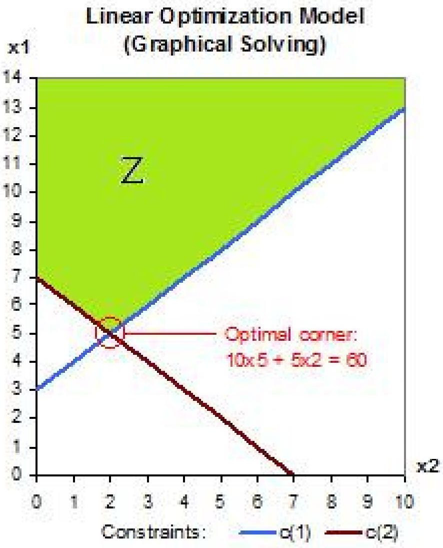
Chart E: Linear vs. Non-Linear Optimization Models

Linear Model:

$$10x_1 + 5x_2 \overset{!}{=} \min$$

$$\begin{aligned} s.d.\ \text{c(1)}:&\ x_1 - x_2 \geq 3 \\ \text{c(2)}:&\ x_1 + x_2 \geq 7 \end{aligned}$$

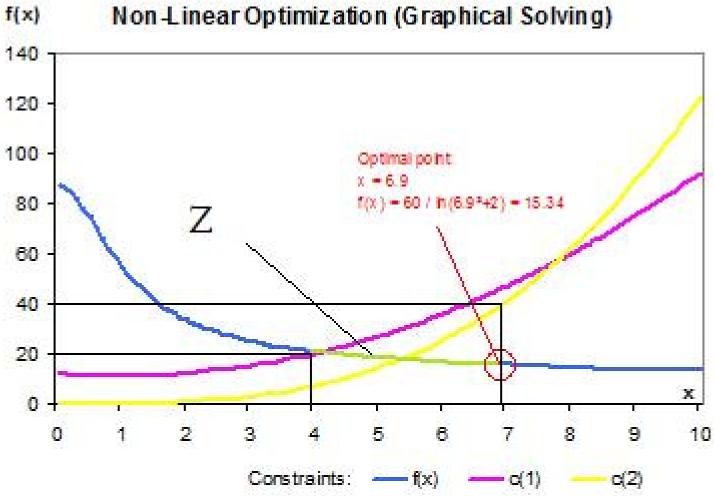
Graphically, the optimal solution of non-linear optimization problems always lies in a corner of the feasible area Z, here colored in green.
In the present case, the minimal value of the target function is obtained for x_1 = 5 and x_2 = 2 (see graph) and equals 60.



Non-Linear Model:

$$\frac{60}{\ln\left(x^2+2\right)} \overset{!}{=} \min$$

$$\begin{aligned} s.d.\ \text{c(1)}:&\ x^2 - 2x + 12 \geq 20 \\ \text{c(2)}:&\ \frac{\left|x^3 - 2x\right|}{8} \leq 40 \\ \text{c(3)}:&\ x \geq 0 \end{aligned}$$



Graphically, the optimal solution is the lowest point on the feasible area Z (i.e. of the green section of the function curve).
In the present case, this point corresponds to x = 6.9, with a 0.1 accuracy.

As described in Chapter 4, this paper presents a non-linear optimization model complemented by an integrality constraint.

3. EFP GmbH: Objectives, Results and Issues

3.1. Company Objectives and Results

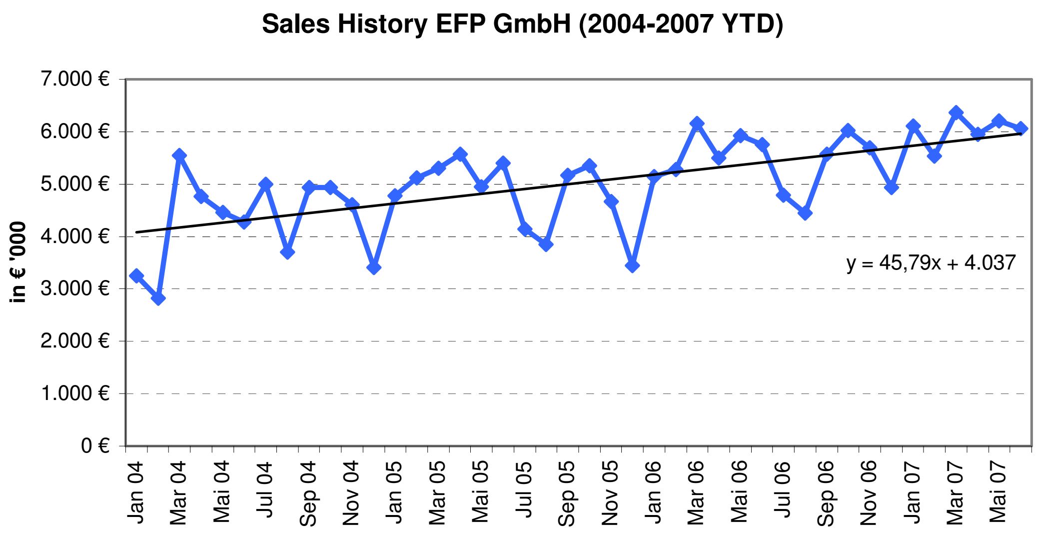
3.1.1. Three Target Measures

There exist three main target measures to evaluate customer service and inventory management at the EFP GmbH. The corporate target[10] as defined in the firm's balanced scorecard is to reach a customer service level[11] of 90% (target n°1). The backorder value should not exceed approximately 6% of sales (target n°2). On the other hand, total inventory days should on average be about 34 days (target n°3). These key measures are calculated monthly, at month end. While the CSL and backorder value are determined for each customer separately, inventory days are computed on the aggregate level of the firm.

3.1.2. Results

As visualized in Chart F, sales have soared consistently over the past three years, with some apparent seasonality shown by low peaks in August and December of each year, and higher sales in March and September. The growth of the company is putting an increasing pressure on existing processes, which need to be redesigned for more efficiency and thus higher productivity[12]. This is particularly important because the factory is currently working at full capacity.

Chart F: Three Year Sales History



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[10] The targets were defined for 2007.

[11] In corporate reporting, the CSL is defined as in equation 2 (see 2.1.4.1).

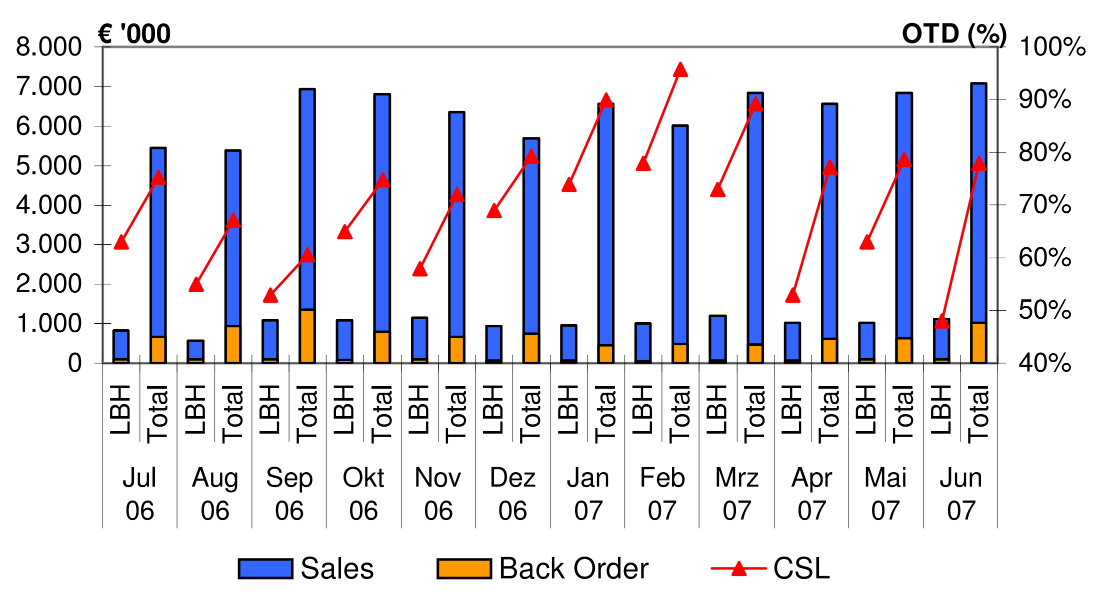
[12] This job is part of the ELSS (EFP Lean and Six Sigma) initiative, which is concerned with redesigning processes and organizing employee trainings for increasing productivity.

The growing order book has caused several organizational issues, presented in the next section, and which produced the following results.

Chart G shows, both in aggregate form for all customers and for the major customer LBH, the sales (value), the back orders (value), and the customer service level over the past twelve months.
From July 2006 to June 2007, back orders represented, on average, 13% of total sales, which is more than twice the target ratio of 6%. For LBH, they represented 10% of sales on average, varying between 6% and 22%, depending on the order value mix (i.e. the value of the late delivered orders relatively to on-time delivered orders).

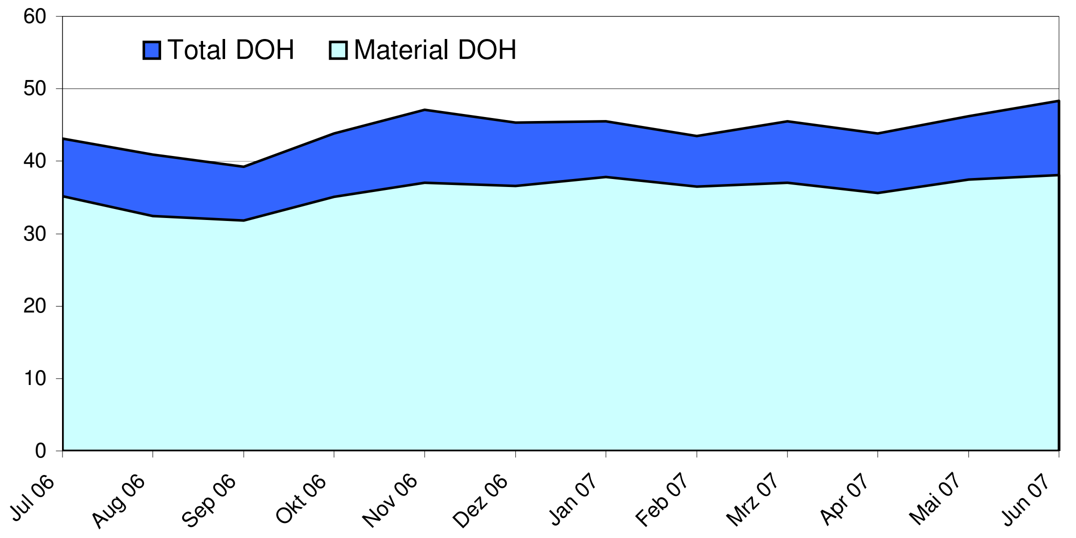
As far as the CSL is concerned, the 90% target has only be reached twice in twelve months, the average being 78%. Customer service for the biggest customer LBH is consistently way lower than the total average, with an average CSL of only 63%, and values ranging between 48% and 78%. This difference is mainly due to the high level of LBH drop-in orders, short-term order entries being one major cause for material stock-outs, as described more in detail in the next section. Computation showed that, over the observed twelve-month period, each increase in the average CSL of LBH of one per cent would have resulted in an increase of the average total CSL of more than 0.25 per cent.

Chart G: Sales, Back-Orders and Customer Service Level



With 44 days, the average inventory days on hand (DOH) over the considered period exceeded the target by 10 days (Chart H). The inventory level appears to be highly correlated to sales. Yet, as the DOH target is annually adjusted to the sales level, the difference to the target is to be seen as a “net” deviation. Work-in-process and finished goods inventories usually represent about 20% of total inventory, and are rather unproblematic as the EFP GmbH operates in a make-to-order production system. Thus, to reach the target, a significant reduction of material inventory is necessary.

Chart H: Material and Total Inventory Days on Hand (DOH)



3.2. Current Issues in Operations and Inventory Management

Several factors have been identified that lower customer service and cause inventories to rise at the same time. Although an exact analysis of each of them would go beyond this paper’s scope, the following observations shall provide an overview of the relevant issues at the EFP GmbH.

Some factors affecting the CSL are measurable with figures and allow a mathematical approach. Regression analyses (see details in Appendix 11), in which the back order value has been set as the dependent and the inventory days and drop-in value as independent variables, showed that:

- inventory days are negatively correlated to the back-order value, and, taken alone, explain 45% of the changes in the dependent variable.
- the drop-in value is positively correlated to the back-order value, and, taken alone, explain 52% of the changes in the dependent variable.

Yet, both factors taken together explain only about 65% of the variation in the back order value. This is due to the fact that they are interrelated for a major part, as security stocks are built to absorb potential drop-ins, i.e. the negative effect of drop-ins is partly offset by high stocks. The remaining 35% are thus due to other factors discussed later in this section, which are difficult to measure and thus impossible to integrate in a multiple regression model.

The first correlation illustrates well the introductive statement of this paper (see 1.1), pointing out the trade-off between high inventory levels and customer service. The regression line equation shows that an increase in inventory days of one day leads to a decrease in back orders of € 61,370 (Appendix 11, Panel C). This relationship is particularly clear under the assumption that a reduction in inventory days is caused in major part by a reduction of security stocks.
The second correlation is also interesting as it shows that each Euro of drop-in causes late deliveries of a value of €1.61. This can be explained by the various negative secondary effects of unplanned orders, starting with a poorly organized production due to frequent short-term scheduling changes.

To complement the overview of causes for back orders, interviews with Management have been conducted and led to several general observations, which are briefly presented here.

According to Management, about 90% of all back orders are either due directly to material stock-outs or to production bottlenecks caused by stock-outs, as the latter avoid timely and lean production planning. The major cause for material stock-outs is poor forecasting, which results in security stocks that are inappropriate with regard to actual short-term order entries (i.e. drop-in). This statement is supported by the results of the multiple regression, which clearly show that drop-in orders turn into back orders, due to the absence of adequate security stocks. Indeed, the current criteria for building a security stock for a component are based on demand volume and the number of customers for which it is needed, but they do not take into account the usual customer order time as compared to the ordering and production lead times (see 2.1.5).
Two causes for material shortage have also been identified that are not related to the forecasting quality. For some critical parts, bottlenecks have been identified in the suppliers’ production systems. In reaction to these bottlenecks, con-

signation inventories are currently built for the concerned parts and alternative suppliers are looked for. Finally, it was pointed out that inaccurate inventory book-keeping was also responsible, even though to a minor extent, for missing material, as components are sometimes taken from inventory without being timely booked out of the system.
Material stock-outs have also a negative secondary effect which leads to a vicious circle of more stock-outs: cross-order material cannibalization. If a hose A requires three parts a, b, and c, and only c is missing, then production planning may schedule the production of hose B in advance, which requires parts a and d, thus using the parts a, which will then miss when parts c arrive.

3.3. Goals and Issues Addressed by the Model

3.3.1. Company Goals Supported by the Model

As the title of this paper already suggests, the model presented in the following chapters is primarily concerned with improving the customer service level for LBH. The relationship between the CSL for LBH and the total CSL, mentioned in section 3.1.2 as 1 : 0.25, shows that reaching a certain CSL for the main customer will have a noteworthy effect on the total CSL as well, i.e. will significantly contribute to reaching target n°1 presented in section 3.1.1.

There is oftentimes a negative correlation between the customer service level and the back-order value[13]. This means that, for a given order value mix, and starting from a given CSL (CSL_0) and back-order value (BO_0), improving the customer service value ($CSL_1 > CSL_0$) will result in reducing the back order value ($BO_1 < BO_0$), thus supporting corporate target n°2 (see 3.1). However, the correlation is only given if the value mix of on-time and late orders is approximately *steady*. To illustrate this relationship, assume that every order has the same value (V). Then each additional per cent of CSL would reduce the back order value by one percent of the total number of orders (TO), times the value of one order (i.e. $TO / 100 * V$). In reality though, the value of an order varies between several euros and several thousands of euros. Thus, it makes a big differences which are the orders delivered late. Indeed, if e.g. the value mix changes such that the late orders become those with the highest value, then an increasing CSL may come with an increase in the back order value.

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[13] For instance, from July 06 to June 07, a correlation factor of -0.46 has been computed between the CSL and the back-order to sales ratio for LBH.

To conclude, on must bear in mind that improving on target n°1 does not always imply an improvement on target n°2, and that the present model does not pretend, for the reasons exposed, to consistently support target n°2.

As for target n°3, it is only addressed indirectly by the model, as the inventory value (which is negatively correlated to the inventory days, assuming constant or rising sales) constitutes the target function to minimize. However, to minimize the inventory costs given a CSL constraint does not mean to try to reach target n°3. Rather, the model is working to reach target n°1 while optimally affecting target n°3. In other words, assuming that everything else stays the same, increasing the CSL for a certain number of parts will cause higher inventories, which goes against target n°3. Yet, it will increase inventory costs optimally, i.e. to the minimal level required for a given CSL.

3.3.2. Issues Which the Model Helps to Solve

The model combines business forecasting with non-linear optimization based on statistical functions. It is designed to better anticipate short-term order entries, i.e. to improve forecasting and optimize security stocks for frequent drop-in products (in this paper only for the main customer LBH). Poor forecasting was shown to be the major cause for material stock-outs, which are responsible for about 90% of all back-orders. Thus, the model shall significantly reduce material stock-outs for the considered products.

Other issues leading to material stock-outs are not addressed by the model and go beyond the topic of this study.

3.4. Information Systems at EFP

The use of a non-integrated and antiquated information system is an overriding cause that hinders proper and accurate operations and inventory management. The corporate wide production system is *MFGPro*, which is mainly used to record and store data, but is quite inflexible when it comes to analyzing. The customization of reports requires a very good knowledge of both the system structure and the programming language. Consequently, the IT department is ultimately the bottleneck when it comes to creating reports.

Whenever possible, data are thus downloaded via text files to an MS Access database. Demand can then be analyzed using the *DAS* (Demand Analysis

System), a program that uses Access to generate a set of standardized reports on a monthly and yearly basis. More customized reports can also be made directly in MS Access. However, data are often simply not available in the form needed for a given analysis, leaving management with estimates or approximate data. This problem has also affected the elaboration of this model, which had to be simplified in some respects, described more in detail in Chapter 4.

The production planning is done using the *DSS* (Demand Scheduling System). This program also uses the data from an Access database, which is based on a daily updated text file from MFGPro. The finished production schedule for the day is then uploaded again to MFGPro every morning.

Finally, there exist two specific applications for the Finance Department. The *Oracle Finance* module is mainly used for book-keeping, whereby most entries are made in MFGPro and uploaded at month-end into Oracle before closing the books. *Encore* is an Excel-based tool with standardized financial analysis reports that are generated on a monthly basis.

The coexistence of several applications based on different MFGPro queries may lead to data inconsistencies. However, the reference data for reporting are the source data from MFGPro. Thus, the input data for this model are also taken directly from the central production system.

4. Model Design, Parameters and Assumptions

4.1. Optimization Model Design

4.1.1. Conceptual Model Definition

Before describing the model using its formal representation (see 2.4.2), the model can be briefly explained in words. For a given number of products (p), the model shall define for each product (i=1 to p) the quantity to be on stock at the beginning of each month (X_i) in order to minimize inventory costs ($f(X_i)$), while maintaining a given level of customer service (CSL_{target})[14].

Using the concepts introduced in the theory part of this paper, $f(X_i)$ is the target function of the model which is to be optimized, i.e. here minimized. The target CSL constitutes a model constraint. The values X_i are the variables that can be adjusted to reach the optimization target under given constraints.

Be c_i the material unit cost of the product i. The inventory cost function $f(X_i)$ is then defined as follows, in accordance with the target function definition of a linear optimization model (see 2.4.2):

(21.) $$f(X_1,...,X_p) = c_1 X_1 + c_2 X_2 + ... + c_p X_p = \sum_{i=1}^{p} c_i X_i \overset{!}{=} \min$$

Be $\hat{Y}_i$ the forecasted demand for product i for the observed time period and SE_i the standard error of the forecast of product i. The relationship between the quantity to be on stock at the beginning of a month and the forecasted demand for that month is then (see confidence intervals under 2.2.4):

(22.) $$X_i = \hat{Y}_i + t_i \cdot SE_i \quad \text{or} \quad t_i = \frac{X_i - \hat{Y}_i}{SE_i} \quad \text{(see equation 10b)}$$

Be α_i the probability of obtaining a t-value greater than t_i in a t-distribution with df degrees of freedom. This probability α_i can be computed using the function $g(t_i\,,\, df)$, integrating the probability density function of t from t_i to +∞ (for more information, see Weisser, 2007):

(23.) $$\alpha_i = P(t_i < t < \infty) = g(t_i, df) = \frac{\Gamma\left(\frac{df+1}{2}\right)}{\sqrt{df \cdot \pi} \cdot \Gamma\left(\frac{df}{2}\right)} \cdot \int_{t_i}^{\infty} \left[\left(1 + \frac{t^2}{df}\right)^{-\frac{df+1}{2}} \right] dt$$

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[14] In the optimization model, the CSL is defined and computed as described under 2.1.4.2, which slightly differs from the CSL as computed for reporting purposes at EFP (see 6.1.2.1).

Relating α_i to the present optimization problem, $g(t_i, df)$ computes the probability of demand exceeding X_i (i.e. what should be on stock at the beginning of a given month). Thus, α_i is the statistical stock-out probability. In the theory chapter, it was already mentioned that, in practice, either t-tables or MS Excel can be used for this computation. Taking the example of a product $i=1$, for which $t_1=0.8755$ and $df=11$, the stock-out probability for this product over the observed period is $\alpha_1= g(0.8755, 11)=20\%$ [15].

Be β_i the probability of not stocking-out of material for product i during the observed month. Then:

$$\beta_i = 1 - \alpha_i \tag{24.}$$

As only drop-in orders are being considered in this model, it is fair to assume that any stock-out will inevitably lead to a late delivery, as a timely material reorder would be impossible (see assumption 4.4.3). Consequently, the *expected* customer service level (CSL_{exp}), as defined in the model (see 2.1.4.2), can be computed by averaging the probability β_i of not stocking out, for $i = 1$ to p:

$$CSL_{\mathrm{exp}} = \frac{1}{p} \cdot \sum_{i=1}^{p} \beta_i \tag{25.}$$

The target customer service level CSL_{target} is to be defined by Management before running the model. Using equation 25, the constraint can then be defined as follows:

$$CSL_{\mathrm{exp}} \geq CSL_{\mathrm{target}} \qquad \Leftrightarrow \qquad \frac{1}{p} \cdot \sum_{i=1}^{p} \beta_i \geq CSL_{t\arg et} \tag{26.}$$

As negative inventories are no valid model results, the variables X_i need to be positive for all products (i.e. for $i=1$ to p). Also, only entire product units can be stored, requiring the quantities to be integers. Thus, only *natural integers* are possible model solutions:

$$X_i \in \mathrm{N} \qquad \text{for } i = 1 \text{ to } p \tag{27.}$$

More constraints may be added to the model. For instance, one will agree that inventory space is not unlimited. Consequently, one may define the available storage volume (SV_{max}, e.g. in cubic meters) as a constraint to the quantities that

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[15] Computed with MS Excel using Function TDIST(0.8755;11;1) (in German, use TVERT)

can be placed on stock. Be v_i the volume in cubic meters of the product i. The storage volume function $h(X_i)$ is then expressed by:

$$\text{(28a.)}\quad h(X_1,\ldots,X_p) = v_1 X_1 + v_2 X_2 + \ldots + v_p X_p = \sum_{i=1}^{p} v_i X_i$$

From there, the subsequent constraint can be derived:

$$\text{(28b.)}\quad \sum_{i=1}^{p} v_i X_i \leq SV_{\max}$$

However, in the present model, the storage volume constraint has, for now, not been implemented, as the EFP GmbH does not possess any record about the volume v of the parts constituting its products. Yet, this constraint may be added at any time if the necessary data are collected.

The model concept as it was implemented at the EFP GmbH can be described in its canonical form as presented in Chart I.

Chart I: Non-Linear Optimization Model – Canonical Definition

$\sum_{i=1}^{p} c_i X_i \overset{!}{=} \min$	→ Linear target function
$s.d.$	
$\frac{1}{p} \cdot \sum_{i=1}^{p} \beta_i \geq CSL_{target}$	→ Non-Linear constraint
$X_i \in \mathrm{N}, \forall i \in [1;p]$	→ Integrality constraint

4.1.2. Model Dataflow

The availability of valid data (see 2.3.2) is a crucial precondition for the model to be applicable. The absence of data about the product parts' volume has for instance caused a model simplification by withdrawing one constraint.

As described in Chart I, the model variables to be optimized are the quantities of each product to be on stock on the first day of the observed month (X_i). They constitute the model output, in accordance with the expected project result (see 1.3). The computation of this output requires a number of inputs.

First, parameters have to be defined for the product selection, which are described in detail in the coming sections. For the selected products (see 4.2), a certain number of values (which constitute the constants of the optimization model) need to be computed: the material unit costs (c_i), the demand forecasts ($\hat{Y}_i$, see 4.3), and the standard errors of the forecasts (SE_i, see 4.4.1).

Additionally, the user must define three model parameters: the degrees of freedom (df), the target customer service level (CSL_{target}), and the model accuracy ($\Delta(\beta)$). These are not attributable to a single product (i), but define the model as a whole. The parameter df depends on the number of data points used to computed the standard errors of the forecasts (see equation 9b), CSL_{target} is the key value to be defined by Management according to corporate objectives, and the model accuracy defines the desired exactness of the optimization and thus the computation time (see 5.3.2).

4.1.3. Model Timing / Data Update Frequency

In order to leave the EFP GmbH enough time to replenish their inventories with the optimal quantities of material, the optimization model must be run in advance. Although most of the materials with long lead times (i.e. over 30 days) are kept on a consignation inventory, it may occur that these stocks are inadequately low, thus causing stock-outs. To allow a timely material planning, it was thus decided with Management to run the model so as to leave 60 days for ordering materials. For example, the inventory optimization for July is done in early May, based on past demand until April.

Demand forecasts are thus updated every month for *three* months ahead: if Y_t is the actual demand for period t, the forecast $\hat{Y}_{t+3}$ for $t+3$ will be made at the beginning of period $t+1$. At the EFP GmbH, material unit costs are constant during one financial year, thus only requiring a yearly update. Finally, the standard errors of the forecasts are updated whenever the forecast method is changed (e.g. because of bias) or when it is periodically reviewed (see 4.3.4).

4.2. Model Product Scope Definition

First of all, it is important to remind that the goal of this model is to improve the customer service level for drop-in products of LBH (see 1.1). Referring to equation 3, drop-in products are those products for which the order time (see definition 2.1.5) is shorter than the time required for ordering materials from suppliers, producing the product and shipping it to the customer. Further analyses confirmed that the low CSL for such products is indeed due to material stock-outs (Chapter 3), i.e. on-time delivery could occur if and only if all necessary material for a product were on stock on the day the customer order comes in.

Which products to include in the optimization model thus depends primarily on the products' order time (OT), material lead time (MLT), production lead time (PLT), and shipping time (ST). It is realistic to consider the MLT, PLT, and ST to be fix over a longer period, as they might only change occasionally e.g. when a new supplier is used (affects MLT), when productivity is increased (PLT), or when incoterms or supplier/customer locations change (MLT/ST). However, the OT can change at each new customer order. Additionally, there is no way to know which will be the OT for a given product at the next order.

Still, the model must focus on the products that are most likely to be ordered with a short OT. Assuming that historical OT are representative of future OT, the *median* OT was identified as the best indicator of a product's most likely OT. As explained in 2.2.1, the median is less sensitive to outliers than the mean, which might distort one's judgment greatly. Take the example of product A, ordered in the past in advance of 2, 4, 5, 5, 7, 9, and 45 days. The mean OT is eleven days, while the median OT is only five days. Assuming that the sum of MLT, PLT and ST is ten days, product A would have been a drop-in product six times out of seven, and should be included in the model. This decision, however, would not have been taken based on the mean, only because the product was once ordered two months (in working days equivalent) in advance.

The relevant MLT for a given product is the MLT of the component with the longest lead time. Materials with long lead times are generally kept on consignation to avoid stock-outs. Provided there is enough material on the consignation stock[16], the MTL of any component used in LBH products varies, with few exceptions, between one and five working days. The PLT and ST are merely the same for all hoses delivered to LBH, i.e. respectively four days and one day. As it would be both very complicated and highly time-consuming to load updated lead times from MFGPro into MS Access on a monthly basis, it was decided in agreement with Management to assume, for every LBH product, a total lead time of ten days (MTL=5, PLT=4, ST=1).

Under this assumption, an LBH drop-in product is thus defined as a product with a median historical order time of ten or less days. Other filtering criteria may be

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[16] This assumption is made for the *selection* of relevant products, but is *no prerequisite* for the model validity. Indeed, the forecast being made three months ahead, the EFP has enough time to reorder even products with lead times up to 60 days.

added in order to appropriately determine which products to select for the model. First, one might want to focus on products that have been ordered at least a certain number of times during the past six/twelve months. Indeed, it is against the optimization objective to build up inventories for products one is not sure to sell anymore. Second, one might want to exclude products from the optimization for which, for some reason, the on-time delivery rate already exceeds a certain level, e.g. 80%. This may be the case of products that have a lead time much shorter than the assumed average of ten days.

The choice of additional filtering criteria is left open to the user, as described in the implementation chapter of this paper (Chapter 5). Chart J summarizes the product criteria that were defined for the model evaluation presented in Chapter 6. The product lists resulting from this process are shown in Appendix 12.

Chart J: Model Product Filtering Criteria

Customer:	**LBH**
Over the past 12 months:	*Threshold:*
Maximal Median Order Time:	**10 days**
Minimal Number of Months Ordered:	**8 months**
Maximal On-Time Delivery Rate:	**80%**

4.3. Demand Forecast Methods

4.3.1. Assessing Different Methods: Test Design

Even without referring to statistical concepts, one will intuitively agree with the statement that the more precise a forecast is, the less security stock is needed, and thus the lower the inventory costs will be. As discussed earlier (see 2.3.3.3), a common measure of forecasting accuracy is the mean absolute deviation (MAD) of the forecast (see equation 17). The lower the MAD, the more precise the forecast, i.e. the lower its standard error (SE). The number of SE one needs to put on stock depends on the defined target CSL, and directly determines the inventory costs (see Chart K). Independently of their accuracy, the selected forecasting methods should not produce biased forecasts for the reasons stated under 2.3.3.3 and illustrated in Appendix 10.

The MAD of seven common forecasting techniques[17] have been computed over a one year test period, from July 2006 to June 2007, for 98 products selected

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[17] The seven methods are listed under 2.3.3.2. As determined under 4.1.3, the methods have been assessed as to their accuracy and bias in delivering forecasts for *three* periods in advance (see equations 13, 14, and 15, as adapted to the model).

for the historical model simulation (Appendix 12). All methods have also been assessed for bias over the same period, using the two bias tests described in section 2.3.3.3 and Appendix 5. The parameters necessary for the use of certain of these methods (i.e. seasonal, trend and smoothing factors) have been previously optimized using the demand data from July 2004 to June 2006.

Chart K: Influence of Forecasting Accuracy on Inventory Costs

Assumptions:
- two SE are necessary to obtain an expected CSL equal to the target CSL
- the material unit costs of the considered product are € 2

Forecast Method A	**Forecast Method B**
- MAD = 20 - SE = 1.25 * 20 = 25 - Security Cost = 25 * 2 = € 50	- MAD = 16 - SE = 1.25 * 16 = 20 - Security Cost = 20 * 2 = € 40

Conclusion:
The smaller the MAD of a forecasting method, the lower are the inventory costs for a given level of customer service (more precisely: a *d*% lower MAD results in *d*% lower costs for a given CSL).

For a given product, the best forecasting technique is the unbiased method with the smallest MAD. Before running the simulation, it was expected that different products would have different best methods. However, it was also expected that one or several methods would perform better than the rest for a significant number of products. For simplicity, this (these) method(s) would then be chosen for implementation in the monthly forecast computation model.

4.3.2. Test Results and Interpretation

As shown in Appendix 7, three methods are consistently outperforming the others as far as accuracy and bias are concerned: simple exponential smoothing, four-month moving averaging, and random walk. For each of the 98 products, one of these methods falls under the two best-performing. Also, there is always at least one of these methods delivering unbiased forecasts.

In accordance with existing literature (Makridakis, 1982 & Armstrong, 1984), the most simple methods have been found to perform best (see 2.3.3.1). All trend-adjusted techniques were on average performing less well than their unadjusted counterparts, which is not surprising when dealing with sub-annual time series (Billah et al., 2005). The autocorrelation analysis (see Appendix 9) shows that, in spite of seasonality of total sales shown earlier in Charts F and G, very few single products are underlying a strong seasonal pattern, justifying the relatively poor performance of the seasonal decomposition technique.

4.3.3. Forecasting Methods Retained for the Model

In view of the results, 1) *simple exponential smoothing*, 2) *four-month moving average*, and 3) *random walk* are the methods retained for computing the forecasts later used in the optimization. The computation is based on the adapted form of equations 13, 14, and 15 (see 2.3.3.2). Though other methods may be more accurate for some products, they are not used for the sake of simplicity.

4.3.4. Periodical Best-Method Allocation

Periodically, at least every six months, the best method out of those three will be determined for each product selected in the model scope. The method allocation is fully automated and follows the principle of focus forecasting (see 2.3.3.4), picking for each product the *unbiased* forecasting technique achieving the *lowest MAD* over the last 12 months. It also allows for manual review and allocation if necessary and appropriate (for more details, see Appendix 13).

4.3.5. On-Going Bias Monitoring

A biased forecast can lead to either constant excess inventory or increase the stock-out probability (see Appendix 10), thus strongly altering the quality and even validity of the optimization model. Additionally to the periodical best method allocation, the forecast techniques will consequently be monitored for bias on a monthly basis using a tracking signal (Hanke / Reitsch, 1995) and a binomial test as described in Appendix 5. If bias is discovered based on those tests, it will be left to the user’s judgment whether the assignment of another forecast method to the product in question may be appropriate.

4.4. Model Assumptions

4.4.1. Relevance of Past Demand Data

As mentioned earlier, the assumption that *past demand pattern are relevant for predicting future demand* is the fundamental postulation made when using quantitative forecasting techniques (see 2.3.3.1).

4.4.2. Normal Distribution of Forecasting Errors Around Zero

As explained earlier in section 2.3.3.3, the fundamental characteristic of an unbiased forecasting technique is that the mean of all forecasting errors is equal to zero over time. This hypothesis is tested by comparing the forecasting error to a tracking signal after each new forecast (see 4.3.5 and Appendix 5). The observation of the histograms of forecasting errors for some sample products (Appendix 14) allows to assume additionally *a normal distribution around that mean*. One characteristic of that distribution is an equal number of positive and negative errors, which is also tested monthly (see 4.3.5 and Appendix 5). The assumption of a normal distribution around zero enables the approximation of the standard error of the forecasting by equation 19b.

4.4.3. Minimum Order Time

In practice, customer pressure may in some instances lead to the acceptance of orders with order times that are shorter than the sum of production lead time and shipping time, i.e. that do not satisfy equation 4. However, the number of such emergencies is kept down to a minimum, and should, according to EFP Management, be completely avoided. For the CSL calculation, it is thus supposed that no *orders are accepted less than five days before due*, corresponding to the sum of the production lead time and shipping time (see 4.2).

4.4.4. Production Lead Time / Shipping Time

Under 4.2, the simplification was made that, for all LBH products, the PLT is equal to four days and the ST equal to one day. From Chapter 3, it appears that there would be merely no production bottlenecks provided there are no material stock-outs. Thus, it is fair to suppose that, *if, and only if, all necessary materials are on stock on the day of the customer order entry, the EFP GmbH will be able to deliver the order on time*. This implies the reciprocal assumption that, whenever material is missing for an order, this order cannot be delivered on time. This is a restrictive assumption, in that an order time may be such that there is enough time for ordering the material, producing and shipping. However, remembering that material stock-outs cause other inefficiencies (see Chapter 3), this simplification stays close to reality.

4.4.5. Month Beginning Dues

The model was designed to determine the optimal quantity of material to have on stock on the first day of a given month for achieving the target CSL. This implies that short-term order entries with due dates within the first four days[18] of that month could not be delivered on time, even if enough material is on stock on the first day. To avoid this, one could decide to put the material on stock five

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[18] PLT + ST = 5 days, see 4.2

days earlier. However, this decision would significantly raise the inventory days on hand which are computed at month end. This negative impact on corporate target n°3 (see Chapter 3) is not justified, as only a small part of the orders is concerned. In addition, a part of those orders is expected to be delivered on time thanks to security stocks and a shorter PLT for certain products. Thus, for the CSL computation, it is assumed that *a due date within the four first days of a month has no effect on the capacity of delivering on-time*.

5. Model Implementation

5.1. Structure of the Integrated System

5.1.1. System Requirements

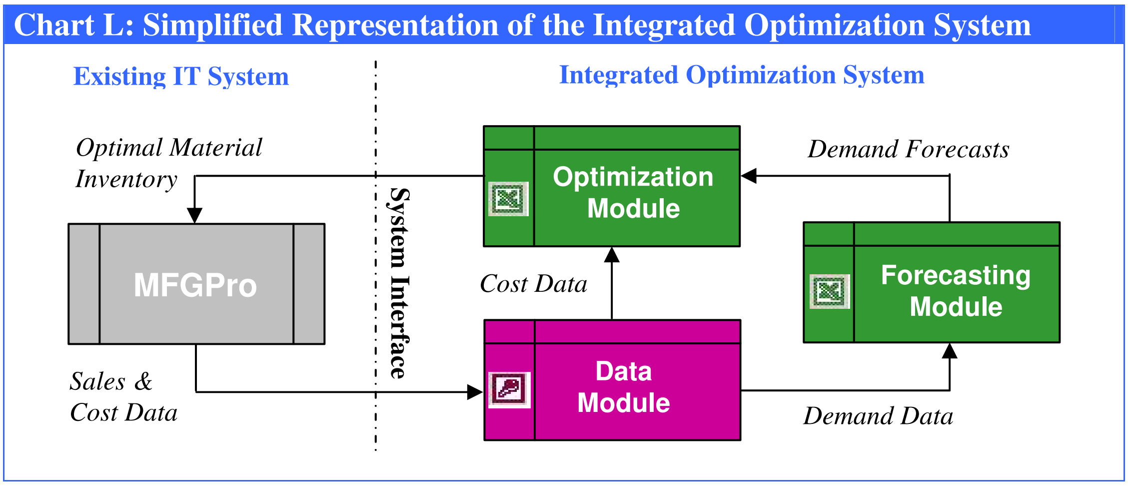
It was established earlier that the model must be run at the beginning of each month (see 4.1.3). In order to be of practical use on a monthly basis, the system needs to be simple to utilize and to maintain, and the results must be quick to obtain and to exploit within the existing IT environment. To satisfy these requirements, an integrated and fully automated optimization system was programmed and implemented. The system structure is described next.

5.1.2. System Structure

A detailed overview of the system structure, visualizing the different system components and the data flows between them, is provided in Appendix 16. This section presents the aggregate system modules and briefly describes how they are interrelated with each other.

On an aggregated level, one could decompose the integrated optimization system in three major components: the Data Module (DM), the Forecasting Module (FM), and the Optimization Module (OM). The system is integrated in the existing IT environment via two interfaces with the MFGPro production system, an input interface and an output interface (Chart L).

Chart L: Simplified Representation of the Integrated Optimization System



The *Data Module* is, for the major part, composed of MS Access Databases that receive sales and cost data for all LBH products in text file format from MFGPro. The product filtering process is then done directly in an MS Access form, where the thresholds on different selection criteria discussed in the previous chapter (see 4.2) can be customized. Demand data are computed and then stored at least for the last 36 months, which is the amount of data required for the forecasting method allocation process (see 5.2.1). Both demand and cost data for the selected products are monthly downloaded to MS Excel files (respectively ‘DemandData’ and ‘CostData’) directly from an MS Access form.

The ‘ForecastData’ Excel file contained in the *Forecasting Module* loads the demand data from the ‘DemandData’ file of the DM. These data are used both for the allocation of the best forecasting technique to each product and for the actual forecast computation, as described more precisely under 5.2.

The forecasts produced in the ‘ForecastData’ file of the FM, as well as the cost data contained in the ‘CostData’ file of the DM, are loaded into the ‘OptimizationModel’ Excel file of the *Optimization Module*, which contains the non-linear optimization algorithm programmed on VBA. The functioning and maintenance of this model is presented in detail under 5.3. The model output, i.e. the quantities of material for each product that are needed on stock on the beginning of the coming month for achieving the desired CSL over that month, can finally be uploaded back into the MFGPro production system. Material orders will then automatically be scheduled according to those quantities.

After this brief overview of the system structure, the two key components of the system, namely the Forecasting Module and the Optimization Module, are presented in greater detail in the following sections.

5.2. The Forecasting Module

5.2.1. Data Flow and Computation

As can be seen from Chart L, the FM transforms the raw demand data input into demand forecasts for three periods ahead of the last known demand (see 4.1.3). Each time that a new product is added to the model *or* that bias is identified for a forecasting technique concerning a given product, *and* at least every six months, the best forecasting technique out of the three retained for the model (namely exponential smoothing, four-month moving average or naïve method, see 4.3.2) needs to be allocated to each product (see 4.3.4).

The best method is determined according to the decision rule defined under 4.3.1, i.e. it is the unbiased method with the lowest MAD. The MAD and bias (see 2.3.3.3) are evaluated based on how each forecasting method would have performed over the past *twelve* months for each product. In order to carry out this test, the FM uses the demand data of the past:

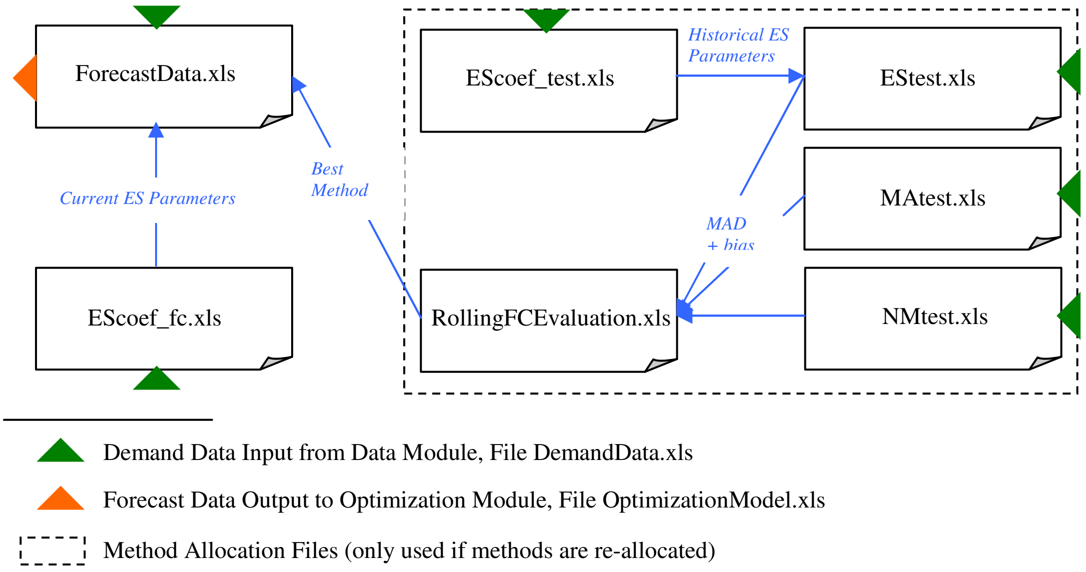
- 36 months for exponential smoothing,
- 17 months for the four-month moving average, and
- 14 months for the naïve method.

The method allocation also works for products that are more recent and thus do not have that many data points. Yet, the best method may then change often from one period to the next as long as no apparent demand pattern can be identified that can be better predicted by the one or the other method.

5.2.2. Module Structure

The structure of the FM module is visualized in Chart M. The central component of the module is the 'ForecastData' Excel file which computes, at the beginning of each month $t+1$, the forecasts for the period $t+3$ (see 4.1.3), using the allocated best method for each product (see 4.3.2). It also evaluates the bias of the currently allocated forecasting technique based on the two tests mentioned in section 4.3.5 and presented in Appendix 5 on a range from 0 to 3, where 0 means no evidence of bias and 3 strong evidence of bias. If there is strong evidence of bias, it is recommended to reallocate forecasting techniques anew.

Chart M: Forecasting Module Structure



In order to compute the forecasts for the products that were allocated the ES-method, the smoothing factor α is optimized using the demand data of the past

24 months in the file ‘EScoef_fc’. This factor stays constant until the forecast methods are being reallocated.

The periodical allocation of the best forecasting technique (see 4.3.4 and 5.2.1) requires five additional files. Three test-files are historically simulating forecasts for each product and computing the MAD and bias (0 to 3) over the past twelve months (i.e. from month M{-12} to month M{-1}). The ES test-file requires the additional input of the smoothing factor α, optimized over the 24 month period *preceding* the simulation period (i.e. from month M{-36} to month M{-13}). The results of each method are compiled in the evaluation file ‘RollingFCEvaluation’, where they are compared to each other, and where the unbiased method (i.e. bias index between 0 and 2) with the lowest MAD is allocated to each product.

Once the methods are allocated, the forecast computation takes place in the file ‘ForecastData’, as described above. Ultimately, the forecasts are transferred to the file ‘OptimizationModel’ in the OM.

5.3. The Optimization Module

5.3.1. Data Flow

The OM is mainly composed of the Excel file ‘OptimizationModel’, which contains the non-linear optimization algorithm described in the next section. The second file in the OM (‘ModelEvaluation’) serves to assess the model validity, as discussed in Chapter 6, but is no active part of the optimization process.

The necessary inputs for the optimization model were already enumerated in section 4.1.2 when defining the model design. From a practical point of view, it has already been established that the product selection is made in the Access Database in the DM, that the cost data were stored in the Excel file ‘CostData’ within the DM, and that the forecasts and standard errors were computed in the Excel file ‘ForecastData’ within the FM. All these data are loaded in the file ‘OptimizationModel’, where the three model parameters (namely df, CSL_{target}, and $\Delta(\beta)$) are then defined by the user before running the algorithm (see 4.1.2).

The following section describes how these input data are used to generate the model output, i.e. the optimal quantity of materials to be on stock at the beginning of the month $t+3$ in order to statistically satisfy the target CSL. This output is ultimately uploaded to EFP’s production system (MFGPro), so that material orders can be planned accordingly.

5.3.2. Non-Linear Optimization Algorithm

In this section, the operating rationale of the optimization algorithm is explained. When appropriate, references are made to the mathematical demonstrations of the equations on which the algorithm is based, presented as Appendices.

To begin with, it can be mathematically demonstrated that, although the model constraint requires an expected CSL higher or equal to the target CSL (see equation 26), the optimal solution must actually satisfy $CSL_{exp} = CSL_{target}$ (Appendix 1). Reducing the feasible area using this reformulated constraint, the simplest feasible solution, which the algorithm uses as starting point for optimization, is to set, for each product, the initial probability of *not* stocking out ($\beta_i = 1 - \alpha_i$) such that:

(29.) $$\text{Initialize: } \forall i \in [1;p] \text{, set } \beta_{\mathrm{i}} = CSL_{\mathrm{target}}$$

Be $\beta_{\mathrm{i(opt)}}$ the optimal probability of not stocking out of product i, and $\Delta_{\mathrm{i}}(\beta) \in \,]0\,;1 - CSL_{target}[$ a small change in β_{i}. Then it can be demonstrated that, if $\Delta_{\mathrm{i}}(\beta)$ tends to zero, there exists an integer $\lambda_{\mathrm{i}} \in Z$ such that (Appendix 2):

(30.) $$\begin{cases} \beta_{\mathrm{i(opt)}} = CSL_{\mathrm{target}} + \lambda_{\mathrm{i}} \cdot \Delta_{\mathrm{i}}(\beta) \\ \text{with } \forall i \in [1;p], \lambda_i \cdot \Delta_i(\beta) \in \left]g\left(\dfrac{-\hat{Y}_i}{SE_i}, df\right) - CSL_{target}\,;1 - CSL_{target}\right[\end{cases}$$

The small change $\Delta_{\mathrm{i}}(\beta)$ is set constant for the each product of the model, and represents the optimization accuracy, noted $\Delta(\beta)$. The smaller $\Delta(\beta)$, the more accurate the computation becomes. Indeed, it is shown that as $\Delta(\beta)$ increases, the model solution departs from the optimum (Appendix 3).

When the algorithm is initialized (equation 29), each $\lambda_{\mathrm{i(t=0)}}$ is equal to zero. In order to satisfy the condition $CSL_{exp} = CSL_{target}$, it can be shown that the sum of all $\lambda_{\mathrm{i(t=}x)}$ for $i = 1$ to p must be equal to 0 after the optimization process (Appendix 2), where x is the total number of operations in the process. In accordance with the zero-sum property, the algorithm is optimizing the integers λ_{i} by "exchanging" small changes $\Delta(\beta)$ among products. All exchanges are done following the financial principle "Buy low, Sell high". One exchange is characterized by two operations: 1) adding ("*buying*") a small change to the product $i \in [1;p]$ with the lowest cost for an additional $\Delta(\beta)$, and 2) removing ("*selling*") a small change from the product $j \in [1;p] - \{ i \}$ with the highest saving for one $\Delta(\beta)$ less.

Thus, the optimum is found when the lowest cost for an additional $\Delta(\beta)$ is higher than the highest saving for removing one $\Delta(\beta)$. In other words, the algorithm stops when no more economically sensible exchange is possible.

To do so, the algorithm computes the cost of adding a small change and the saving from removing a small change after each exchange anew. If $r(\alpha_i, df)$[19] is the function returning the t-value t_i corresponding to a stocking-out probability α_i for df degrees of freedom, then the costs C_i $(\Delta(\beta))$ and savings $S_i(\Delta(\beta))$ for each product are computed as follows (demonstration in Appendix 3):

(31a.) $$C_i(\Delta(\beta)) = SE_i \cdot c_i \cdot \left[r((1-(\beta_i + \Delta(\beta))), df) - t_i\right]$$

(31b.) $$S_i(\Delta(\beta)) = -SE_i \cdot c_i \cdot \left[r((1-(\beta_i + \Delta(\beta))), df) - t_i\right]$$

The total number of operations x can be calculated as follows:

(32.) $$x = \sum_{i=1}^{p} |\lambda_i|$$

From equation 30, one can see that given $\beta_{\mathrm{i(opt)}}$ and CSL_{target}, λ_{i} increases as $\Delta(\beta)$ becomes smaller. Hence, with equation 32, the number of operations x and thus the computation time increase as the model accuracy increases.

Once the globally optimal solution $X_{\mathrm{i(opt)}}$ is computed such that for $i=1$ to p, $X_{\mathrm{i(opt)}} \in \mathrm{R}^+$, the decimal quantities are optimally converted into integers to satisfy the integrality constraint (equation 27). More details are provided in Appendix 4. A screenshot of the optimization model user interface is provided in Appendix 19.

5.4. User Interface and Maintenance

5.4.1. Working with the System

As defined in the system requirements (see 5.1.1), a fully automated system has been programmed to enable an easy utilization, achieve quick results and avoid errors. The system is composed of a series of sequentially interrelated MS Access Databases and MS Excel Files. The data transfer from one file to the next is automated and is launched via a user form popping up when opening a file (see examples in Appendix 17). All operations to be done within a file are also controlled via user forms. Additionally, each file contains a detailed help form were each step is explained in detail and where the user can learn

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[19] This function corresponds to the TINV function of MS Excel.

more about the statistical concepts and forecasting techniques (see 2.2 and 2.3) he or she is using. A user's handbook[20] has been written for the EFP GmbH, which provides detailed guidance about the procedure that needs to be done monthly, or, for certain activities (e.g. method allocation), periodically.

The optimization process time from the downloading of the raw data out of MFGPro to the uploading of optimized material inventory quantities back to the production system has been measured as presented in Chart N.

Chart N: Optimization Process Time

The process only requires one employee. The times are indicated as follows:

Effective User Operating time ↗ 5 min (10) ↖ Forecasting and Optimization time for 100 products

The effective user operating time is the time the user is actively executing the process. The forecasting and optimization time is the time required for the computation of the forecasts and then the optimization (in the example, 10 min per 100 products). During this time, which is indicated to the user in a status bar, the user may complete other tasks in parallel.

	Simple Procedure	Extended Procedure
Familiar	5 min (10)	20 min (15)
Not Familiar	20 min (10)	40 min (15)

Note:
An employee is said to be familiar with the process when he does not need to consult the handbook or help forms anymore.

The simple procedure to be done monthly consists of updating demand data, computing forecasts, and optimizing inventories for the products currently selected in the model, using current cost data and forecasting methods. The extended procedure additionally comprehends a new product selection, a cost update, and a new forecasting method allocation. Although the extended procedure may also be done monthly, the user may choose to do it quarterly or every half-year, depending on the degree of reactivity he desires with regard to changes in product characteristics and bias.

5.4.2. Maintaining the System

The monthly and periodical update of data (i.e. demand, forecast, cost) is part of the optimization process itself. Maintenance as such is reduced to the occasional deleting of old data from the MS Access Database (as recommended: older than 40 months) in order to limit the database size and thus reduce the operating time. Also, the validity of the model itself should be monitored annually, using the method described in Chapter 6.

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[20] An extract of the User's Handbook is attached in Appendix 18.

5.5. System Limitations – Areas for Improvement

5.5.1. Constraints Limitation

The optimization algorithm has been programmed based on the non-linear optimization model as defined in Chapter 4 (see Chart I). Any additional constraints to be added (such as the storage volume constraint, see equation 28b) would require a prior upgrade of the algorithm.

5.5.2. Variables Limitation

To avoid excessive computation times, the algorithm has been limited to 2000 variables, i.e. 2000 different products can be selected for the optimization. In total, about 3500 different products have been sold to LBH in the past two years. The model is, by design, only sensible for drop-in products (with a median order time of maximum ten days) which are ordered several times a year (see 4.2.). This is the case of about 10% of those items. Thus, this limitation has actually no practical consequences if limiting the model to LBH products.

5.5.3. Customer Limitation

For now, the model was designed and implemented for LBH products exclusively, as initially defined by Management in the project requirements. This makes sense in that LBH is the biggest customer on the one hand, and the customer with the highest proportion of drop-in products on the other hand. Yet, the effect on the total CSL (which is already significant, see 3.1.2) could be further increased by extending the model scope to all drop-in products. Both the forecasting and optimization modules would not require any changes for this extension. However, the raw data would need to be filtered directly in MFGPro, as the currently downloaded text file does not contain all necessary information about all customers. Also, one must bear in mind the potential constraint of the limitation of the number of variables to 2000 (see 5.5.2).

6. Model Evaluation and Validity Monitoring

6.1. Model Evaluation

The model evaluation is based on the same assumptions than the model itself (see 4.4). The model validity is assessed with a statistical hypothesis test (see next section) run over the test period from July 2006 to June 2007. Then, the real CSL is compared with the historically simulated CSL that would have been achieved using the model over the same period. Unfortunately, the real inventory costs corresponding to the materials for the products of the model cannot be determined historically from the EFP's information systems. Instead, a simulation made with the optimization model was used to determine the average cost/service function. From this function, an estimate of the inventory costs of capital (see equation 1) with and without using the model.

6.1.1. Statistical Hypothesis Test

The five steps of a hypothesis test are described in Chart D, section 2.2.5. This same structure is followed here.

The hypothesis that needs to be tested here is whether the average customer service level achieved over time using the model is equal to the target customer service level that the user has defined (here $CSL_{\text{target}} = 80\%$). Thus:

(33.) $$H_0 : \mu_{CSL} = CSL_{target} \text{ , and } H_1 : \mu_{CSL} \neq CSL_{target}$$

where μ_{CSL} is the mean customer service level of the population (i.e. the mean CSL over an unlimited period of time in the future)

The model validity shall be tested at a confidence level of 90%, i.e. a 10% alpha-error of rejecting the null hypothesis (H_0) although it is true is accepted. As the hypothesis is assessed using a small sample of twelve data points and the population standard deviation is unknown, a t-distribution with eleven degrees of freedom is used rather than a normal distribution (see 2.2.2). Thus:

(34.) $$\alpha = 0.10 \text{ , and } t_{crit} = t_{(\alpha=0.1;df=11)} \approx \pm 1.8$$

Consequently, the null hypothesis will be rejected if the sample statistic (i.e. the average customer service level obtained with the model over the twelve-months period from July 2006 to June 2007) is smaller then –1.8 or greater than 1.8.

Be t_{sample} this sample statistic. Then the decision rule can be written:

$$\text{(35.)} \quad \text{Reject } H_0 \text{ if } t_{sample} \notin [-1.8; 1.8]$$

The period July 2006 to June 2007 is the only time frame available for performing the hypothesis test at this point in time, for historical demand data necessary for the forecast computations are not available for prior periods. Also, this is a first time assessment of a hypothesis about the mean CSL of an infinitely growing population (one data point is added each month). Thus, the “sample” period actually corresponds to the whole population at this point in time, but inferences about the model validity shall be made for the future (i.e. for a larger population). Though this sample is not clearly random, there is no sampling error either: it is simply the first sample of twelve data points that can be made out of the population to be observed. It is however important to monitor the model validity regularly (e.g. annually) by then picking *randomly* for instance twelve months among all past months (more about this procedure under 6.2).

Chart O summarizes the results of the historical simulation (more information about this simulation is provided in Appendix 20), i.e. it lists the CSL that would have been obtained in each month from July 2006 to June 2007 if the model had been used. Using equation 5b, we compute the mean CSL of the sample:

$$\text{(36.)} \quad \overline{CSL} = \frac{1}{n} \cdot \sum_{i=1}^{n} CSL_i \text{ , here: } \overline{CSL} = 0.8141 \text{ (see Chart O)}$$

Using equation 6b, we also calculate the standard deviation of the sample:

$$\text{(37.)} \quad s = \sqrt{\frac{\sum_{i=1}^{n} (CSL_i - \overline{CSL})^2}{n-1}} \text{ , here: } s = 0.0861 \text{ (see Chart O)}$$

Chart O: Simulated CSL from July 2006 to June 2007

Month	Sim CSL	Month	Sim CSL
06-07	0.8571	07-01	0.8113
06-08	0.9388	07-02	0.8679
06-09	0.8265	07-03	0.8396
06-10	0.7245	07-04	0.8774
06-11	0.6122	07-05	0.8396
06-12	0.7347	07-06	0.8396

Mean	**0.8141**
Std Dev	**0.0861**

The sample statistic t_{sample} is then simply computed by applying equation 11b to equation 10b (the standard deviation of the sample is used to approximate the unknown standard deviation of the population):

$$\textbf{(38.)} \quad t_{sample} = \frac{\overline{CSL} - \mu_{CSL}}{\frac{s}{\sqrt{n}}}, \text{ here: } t_{sample} = \frac{0.8141 - 0.8}{\frac{0.0861}{\sqrt{12}}} = 0.5673 \quad \text{(see equations 36/37)}$$

Assessing the sample statistic with regard to the decision rule (i.e. $0.5673 \in [-1.8; 1.8]$, see equation 35), the null hypothesis cannot be rejected based on this sample evidence. In other words, *the validity of the model is not called into question* based on the first twelve months data.

A hypothesis test does not allow to draw the conclusion that the null hypothesis is true (Hanke / Reitsch, 1995) when it cannot be rejected. The model appears to be valid, but a regular assessment of its validity is recommended (see 6.2).

6.1.2. Historical Comparison of Real and Simulated Data

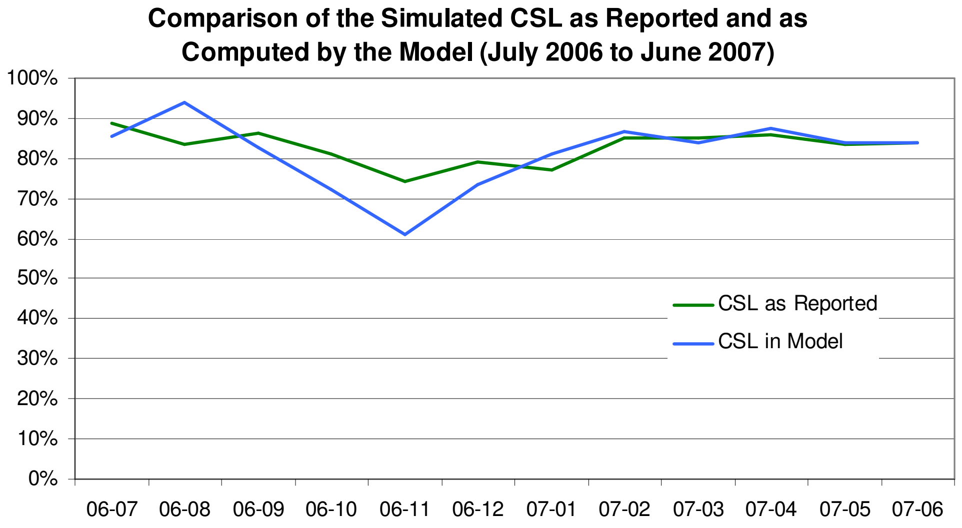
6.1.2.1. Reported CSL vs. Model Adapted CSL

As discussed in section 2.1.4, the CSL computation for corporate reporting differs from the CSL computation done by the model. In the model, for each product, the total monthly demand is considered as one single order (see 2.1.4.2). This assumption is necessary for the optimization process, given that it cannot be foreseen on how many orders the demand is split in a given month. However, as was defined early on, the CSL as reported on EFP's balanced scorecard is computed on order basis, i.e. it represents the percentage of orders timely delivered (see 2.1.4.1).

The simulated CSL values of Chart O were computed according to the model adapted definition (see 2.1.4.2), as the validity of the model must be assessed using values computed in the same fashion than the model does for optimizing (see blue line in Chart P). However, it is of interest to verify, beyond the model validity, if the so obtained CSL values are approximately close to the values that will ultimately be reported. Indeed, when Management indicate the customer service level target as a model parameter, they have in mind the reporting definition and not the adapted model definition.

The results of the historical simulation (plotting comparatively the customer service levels that would have been obtained according to each of the two definitions) show that the differences that may arise between both approaches in single periods offset each other over time (see Chart P). The reported CSL is even less subject to variation around the target CSL here defined as 80%. Thus, the difference of definition between reporting and model is irrelevant in practice.

Chart P: Difference Between Reported and Adapted Model CSL

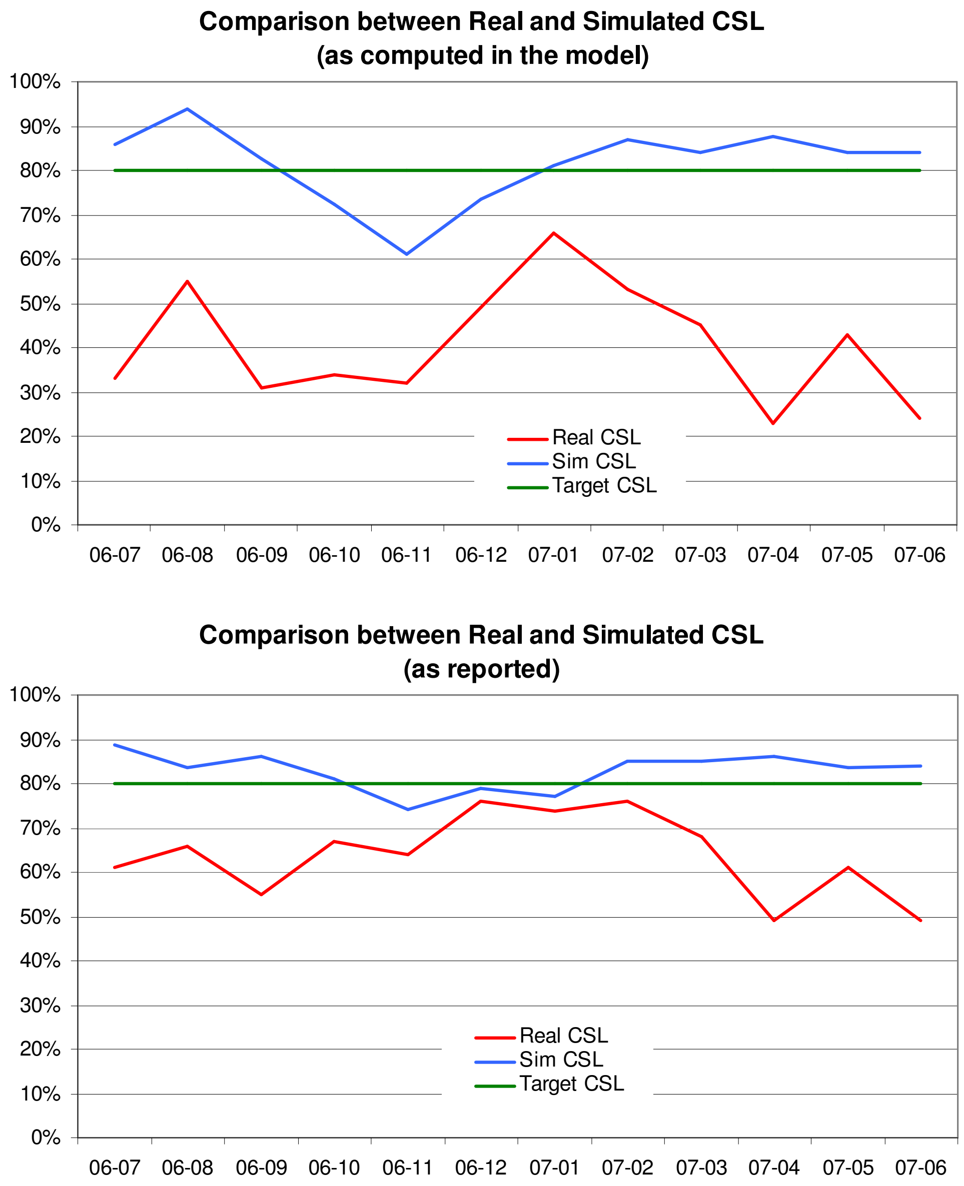


6.1.2.2. Real CSL vs. Simulated CSL

For comparing the real CSL (i.e. the CSL that has been obtained in reality for the observed products over a given period) with the simulated CSL (i.e. the CSL that would have been obtained for the observed products over a given period), it is important to use the same definition for each value. To do so, both the real and the simulated CSL have been computed according to both CSL definitions discussed in the prior section, and plotted on two graphs (see Chart Q).

In both cases, the model would have helped the EFP GmbH to achieve a significantly higher CSL than the one actually obtained. The difference is the greatest when assuming that the total monthly demand for one product corresponds to a single order (then the model outperforms current results by 41% on average). Indeed, while this total-demand-approach is the focus of the model, EFP concentrates on its own definition, i.e. on delivering orders on-time. Yet, even then, the model maintains a healthy advance of 19% on average, with much steadier results on top. This analysis also shows that, whereas the focus on the on-time delivery of total demand triggers similar levels of customer service in the order approach (see 6.1.2.1 and Chart P), the focus on the on-time delivery of single orders does not at all imply similar levels of customer service when considering the total monthly demand (red lines in Chart Q).

Chart Q: Historical CSL Comparison



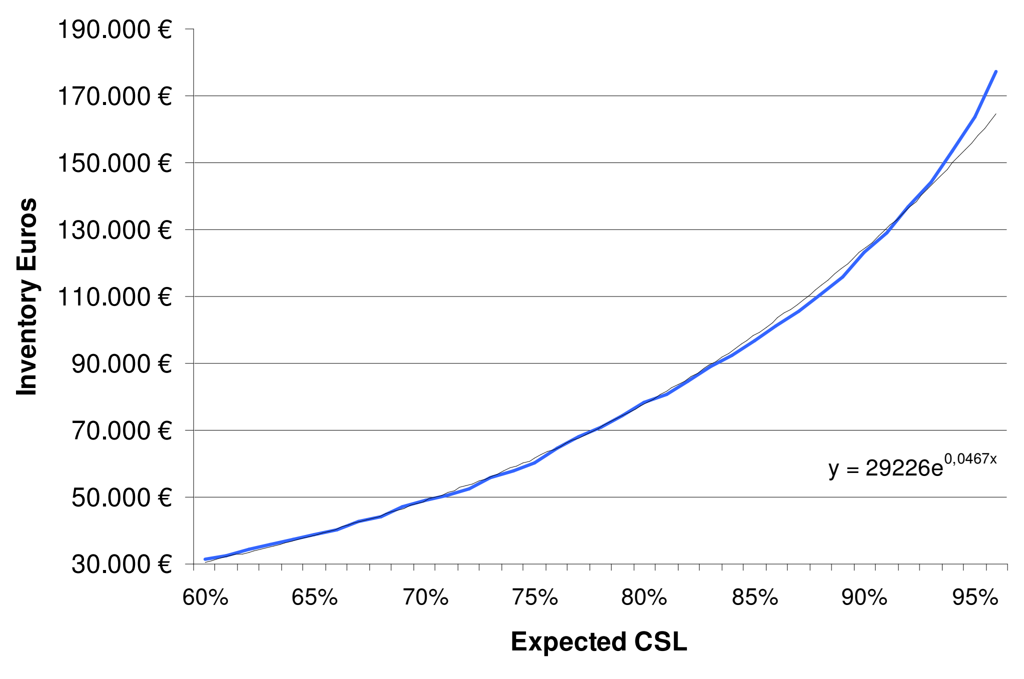
6.1.2.3. A higher CSL: at what cost?

It has been established that the model appears valid in that its output quantities produce, on average, the defined target CSL (see 6.1.1), be this target measure defined by reporting standards or in the model focus (see 6.1.2.1). The prior section has shown that, using the model, a significantly higher level of customer service could have been achieved over the past year. However, it is well known from the early stated dilemma (see 1.1) that a higher CSL generally comes at a cost. The purpose of this section is to estimate, if any, the additional ICC incurred, corresponding to the increase in CSL that could have been achieved.

The information systems at EFP do unfortunately not allow to retrieve the material inventory value at the beginning and at the end of each period for the LBH products selected for the model. As these data would be necessary to compute the real ICC (see equation 1), real ICC data are not available.

However, an estimation can be made by extrapolating the cost/service curve obtained from a simulation in which the optimization was run for different target CSL, between 60 and 96% (see Chart R), for each month of the test period.

Chart R: Average Monthly Material Investment / Service Curve (June 06 - July 07)



This graph visualizes the necessary average material investment (i.e. inventory at the beginning of a month) for assuring a given level of CS. The curve is exponential and can be approximated by the equation displayed above. Its shape can vary monthly according to the number of products considered in the model and their material costs, the demand forecasts, and the respective standard errors. Here presented is the curve of average costs over the considered test period.

The simulation results presented in detail in Appendix 21 show that the average percentage (in value) of the initial material that would be sold over the month is 42%. Assuming a yearly capital return rate of R=20%, the annual ICC that would have been incurred using the model with a target CSL of 80% is, applying equation 1:

$$\textbf{(39.)}\quad ICC_{CSL=80\%} = 20\% \cdot \left(78{,}080 - \frac{42\% \cdot 78{,}080}{2}\right) = €12{,}338$$

It was established above that, when using the model, the achieved average customer service levels computed using the reporting and the model adapted definition are fairly equal over time (see 6.1.6.2). Assume Management had defined a target CSL of 60% as the optimization constraint. The model being valid (see 6.1.1), a CSL of about 60% would then have been achieved over the test period (i.e. a value that is close to the real reported CSL of 63.83%, see Appendix 20), independently on the CSL definition.

Assuming that the average portion of the initial material stock sold over the month stays constant at 42% independently of the target CSL, the annual ICC corresponding to a target CSL of 60% can be computed using the corresponding material investment from the cost/service (i.e. € 31,278) curve in equation 1:

$$\textbf{(40.)}\quad ICC_{CSL=60\%} = 20\% \cdot \left(31{,}278 - \frac{42\% \cdot 31{,}278}{2}\right) = €4{,}942$$

However, this value corresponds to the minimal ICC under optimized conditions (i.e. ordering the optimal quantities defined by the model). For producing a realistic estimate of the real average monthly ICC over the period, additional costs need to be added. These comprehend first of all the costs caused by the random (i.e. not dependant on standard errors and material costs) determination of (security) stocks, leading to a sub-optimal allocation of resources in comparison to the model optimum. A Monte-Carlo simulation with 50 iterations, in which material quantities were randomly defined such that the expected CSL is equal to 60%, produced an average monthly initial material investment of € 124,298 [21] (Appendix 22), i.e. way more than the optimized cost for a target CSL of 80%.

Additionally, one must bear in mind all the side effect costs of material stock-outs, such as costs from production inefficiencies, express shipping costs, etc. (see Chapter 3). Although part of these costs would subsist even when using the model (as for a target CSL of 80%, still 20% of stock-outs are accepted), they would be much reduced.

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[21] The simulation was made for the month July 2006.

To sum up, the real level of customer service obtained so far (i.e. 63.83%) could have been achieved for almost € 7400 less in ICC (see equations 40 and 39). Put differently, it is a founded guess to assume that, for approximately the same or even less than the current real costs, a CSL of 80% can be achieved thanks to the model.

6.2. Model Validity Monitoring

6.2.1. Annual Validity Hypothesis Test

The hypothesis test (see 6.1.1) demonstrated that the validity of the model is not called into question. Yet, this is not enough to assure the model will be valid indefinitely. A regular (at least annually) monitoring is thus required. To do so, a hypothesis test similar to the one described above can be conducted using the 'ModelEvaluation' Excel File. This file automatically performs the test when given the following input:

- the null hypothesis (i.e. here the target CSL)
- the level of significance (allowed alpha-error) , and
- the simulated CSL obtained in twelve *randomly* selected months

The selection is to be made among all the months between July 2006 and the last month for which the model has been used when the validity hypothesis test is to be conducted. The 'ModelEvaluation' file contains a randomizing function that can do a random selection of months for the user.

Each month, when uploading the optimized material quantities from the 'OptimizationModel' file to MFGPro, these data are also transferred to a storage sheet in the 'ModelEvaluation' file. For the twelve months randomly selected, the user will have to load the real demand for the products then in the model scope either from the MS Access Database or directly from MFGPro. Optimized quantities and real demand are then compared to compute the simulated CSL for each month. The twelve data points so obtained constitute the random sample used to perform the hypothesis test. If, based on the test, the model validity is called into question (which is not expected though), the model design would have to be reviewed.

6.2.2. Practical Validity / Applicability Test

Besides the fundamental validity of the model as designed, the practical validity (i.e. relevance) is also important. Indeed, as in every model, simplifying assumptions had to be made that can have significant effects on the real impact the model may have on the CSL. For example, a central assumption of the model is that, if and only if the material is on hand on the day of an order entry, this order will be delivered on time. However, in reality, there might be other factors that impede on-time delivery (e.g. a production bottleneck), or that enable on-time delivery even if the material was not there (e.g. shorter material lead times). Also, cross-product material cannibalization (see 3.2) may remain a problem. Indeed, the model is concerned with an isolated number of products. However, the materials used for these products are also used for other products not being considered. Thus, if part of the material planned for products within the model scope is used for products out of the scope, then the CSL achieved for the products of the model will dump. To prevent this, the “reservation” of material could be a solution. However, the negative effect of reservations for orders not yet entered is that, if demand is ultimately under forecast, the material stays unused on stock while other products are delivered late. Which way to go is, in any case, still a Management decision, the present model only constituting a tool to support it.

7. Conclusion and Critical Outlook

The model presented in this study has been designed to the particular needs and data constraints of the EFP GmbH. It determines, for a definable set of critical LBH products, the optimal quantities of material to have on stock of the first day of the month $t+3$ (t being the month of last known demand), thus leaving 60 days for replenishing the stock accordingly (Chapter 4).

As demanded, the model has been implemented in a totally automated information system integrated in the current IT environment of the company. This system is easy to use and produces results relatively quickly, requiring five to forty minutes a month of active handling and no more than fifteen minutes of computation time per one hundred products to be considered (Chapter 5).

If applied consistently, the model is expected to help the EFP GmbH to increase its customer service level for critical (drop-in) products ordered by its biggest customer LBH by 20% to 30% on average, without increasing total costs. The CSL for LBH may consequently, depending on how many products are considered in the model, increase by 10 to 20% (Chapter 6). Knowing the 1 : 0.25 relationship between the total CSL and the LBH CSL, the global impact of the model the EFP's CSL is expected to be an average increase of up to 4%. Besides, the model scope may be extended from LBH drop-in products to all drop-in products, in order to increases the global impact on the CSL (Chapter 3).

However, it must be highlighted that the model is based on several simplifying assumptions that might, to varying extents, differ from real conditions. While the model is focusing on the issue of unplanned drop-in orders as a major cause of material stock-outs, it is not concerned with adjacent problems that were identified as factors for late delivery. In particular, it will be important for the EFP GmbH to look for new suppliers in instances where the existing ones have reached their capacity limit, and to extend the range and quantity of products kept on consignation to enable a higher flexibility in managing operations.

Also, it must be kept in mind that the goal of the present paper is to improve the CSL as defined by the EFP GmbH. Related goals such as reducing the back order value and reducing inventory days did not lie in the project focus.

VII. Literature Review and Other Sources

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VIII. Disclaimer

I hereby certify that this material, which I submit for assessment on the program of study leading to the award of MSc in International Management is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

August 31st, 2007

Stefan Gebauer

IX. Appendices

Important Note:

The real product names (i.e. references) have been replaced by fake references for confidentiality. However, the replacements are consistent (i.e. each real reference has been attributed the same fake reference throughout the Appendices), and all figures are real data.

In some Appendices, the number format is German, i.e. a point is a digits separator and a coma separates the integral from the decimal part.

Appendix 1: The Optimal Solution satisfies $CSL_{exp} = CSL_{target}$

Demonstration:

Demonstrate that, for any expected customer service level superior to the target customer service level, the inventory costs are higher than then when the expected customer service level equals the target customer service level.

Note:

df	degrees of freedom
X_i	quantity of product i to be on stock on the beginning of the observed month
Y_i	demand forecast for product i
SE_i	standard error of the forecast for product i
c_i	material unit costs of product i

Be α_i the probability of stocking out of material for product i in the observed period. Then, by definition, we have (see 4.1.1)[22]:

$$\alpha_i = P(t_i < t < \infty) = g(t_i, df) = \frac{\Gamma\left(\frac{df+1}{2}\right)}{\sqrt{df\cdot\pi}\cdot\Gamma\left(\frac{df}{2}\right)} \cdot \int_{t_i}^{\infty}\left[\left(1+\frac{t^2}{df}\right)^{-\frac{df+1}{2}}\right]dt \tag{1}$$

Be β_i the probability of having enough material on stock for covering the demand for product i over the observed month. Then we have:

$$\beta_i = 1 - \alpha_i \tag{2}$$

From equations (1) and (2), we deduce that β_i is positively related to t_i, i.e. that β_i increases if t_i increases.

We already know that (see 2.2.2):

$$t_i = \frac{X_i - \hat{Y}_i}{SE_i} \tag{3}$$

From equation (3), we see that t_i is positively related to X_i, i.e. that t_i increases if X_i increases. Be CSL_{exp} the expected customer service level for the observed month:

$$CSL_{\exp} = \frac{1}{p}\cdot\sum_{i=1}^{p}\beta_i \tag{4}$$

Thus, if we define $\Delta_i(\beta)$ as a small change in β_i, and $\Delta(CSL_{exp})$ as the corresponding change in the expected level of customer service, we obtain:

$$\Delta(CSL_{\exp}) = \frac{1}{p}\Delta_i(\beta) \tag{5}$$

---

[22] This function corresponds to the TDIST function of MS Excel (TVERT in German).

Equation (5) implies:

$$\textbf{(6)}\quad \begin{cases} \Delta(CSL_{\exp}) > 0 \Rightarrow \sum_{i=1}^{p} \Delta_i(\beta) > 0 \\ \Delta(CSL_{\exp}) < 0 \Rightarrow \sum_{i=1}^{p} \Delta_i(\beta) < 0 \end{cases}$$

The material unit costs of product i are always strictly positive, i.e. $c_i > 0$. Be $\Delta_i(X)$ a small change in X_i, and $C(\Delta_i(X))$ the material cost of the change. Then, we have:

$$\textbf{(7)}\qquad \text{Given } c_i > 0,\ \begin{cases} \Delta_i(X) > 0 \Rightarrow C(\Delta_i(X)) > 0 \\ \Delta_i(X) < 0 \Rightarrow C(\Delta_i(X)) < 0 \end{cases}$$

Be $\Delta(\sum_{i=1}^{p} c_i X_i)$ the change in total inventory costs corresponding to the change $\Delta(CSL_{\exp})$ in the expected customer service level.

From the above, we conclude that:

(8a)

$$\Delta(CSL_{\exp}) > 0 \Rightarrow \sum_{i=1}^{p} \Delta_i(\beta) > 0 \Rightarrow \sum_{i=1}^{p} \Delta_i(X) > 0 \Rightarrow \sum_{i=1}^{p} C(\Delta_i(X)) > 0 \Rightarrow \Delta(\sum_{i=1}^{p} c_i X_i) > 0$$

(8b)

$$\Delta(CSL_{\exp}) < 0 \Rightarrow \sum_{i=1}^{p} \Delta_i(\beta) < 0 \Rightarrow \sum_{i=1}^{p} \Delta_i(X) < 0 \Rightarrow \sum_{i=1}^{p} C(\Delta_i(X)) < 0 \Rightarrow \Delta(\sum_{i=1}^{p} c_i X_i) < 0$$

Assume CSL_{exp} is such that $1 > CSL_{exp} > CSL_{target}$.

Be $\Delta(CSL_{\exp})$ the change in the expected customer service level such that $CSL_{exp} = CSL_{target}$.

Then this change is negative, as we need to reduce CSL_{exp}:

$$\Delta(CSL_{\exp}) = CSL_{t\arg et} - CSL_{\exp} \Rightarrow \Delta(CSL_{\exp}) < 0 \Rightarrow \Delta(\sum_{i=1}^{p} c_i X_i) < 0$$

<u>Conclusion</u>: from (8b) we see that any possible reduction in CSL_{exp} leads to cost savings. Thus, any $CSL_{exp} > CSL_{target}$ is not cost-optimal, QED.

Appendix 2: The optimal probability of not stocking out $\beta_{i(opt)}$ is an integral number of small changes away from CSL_{target}

Demonstration:

a) Show that if, for each product $i \in [1;p]$, the probability β_i of not stocking out is set equal to the target customer service level CSL_{target}, the solution belongs to the feasible area.

b) Be $\Delta_i(\beta)$ a small change in the probability β_i of not stocking out of product i in the observed month. Demonstrate that, for each product i, the optimal probability $\beta_{i(opt)}$ of not stocking out is an integral number λ_i of small changes $\Delta_i(\beta)$ away from the target customer service level CSL_{target}.

c) Show that, when the optimal solution is obtained, the sum of the integers λ_i for i=1 to p is equal to zero.

Note:

df	degrees of freedom
p	number of products in the model
X_i	quantity of product i to be on stock on the beginning of the observed month
Y_i	demand forecast for product i
SE_i	standard error of the forecast for product i
c_i	material unit costs of product I

Part a)

It was shown in Appendix 1 that the optimal solution must satisfy $CSL_{exp} = CSL_{target}$. Let's set the probability of not stocking out β_i of each product equal to CSL_{target}.

$$\textbf{(1)} \qquad \forall i \in [1; p], \beta_i = CSL_{target}$$

We know that:

$$\textbf{(2)} \qquad CSL_{\exp} = \frac{1}{p} \cdot \sum_{i=1}^{p} \beta_i$$

Thus, with (1) in (2):

$$\textbf{(3)} \qquad CSL_{\exp} = \frac{1}{p} \cdot p \cdot CSL_{target} = CSL_{target} \qquad \textbf{(QED)}$$

Conclusion: Setting all β_i equal to CSL_{target} is a feasible solution with regard to the optimality constraint demonstrated in Appendix 1.

Part b)

If $\beta_{i(opt)}$ is the optimal probability of not stocking out of material for product i, then there exists a residual real number $\gamma_i \in \Re$ such that:

$$\textbf{(4)} \qquad \beta_{i(opt)} = CSL_{target} + \gamma_i \text{, with } \forall i \in [1; p], \gamma_i \in \left]g\left(\frac{-\hat{Y}_i}{SE_i}, df\right) - CSL_{target} ; 1 - CSL_{target}\right[$$

where $g\left(\frac{-\hat{Y}_i}{SE_i}, df\right)$ is the probability of stocking out when $X_i = 0$ (minimum probability).

Be $\lambda_i \in \mathrm{Z}$ an integer and be $\Delta_i(\beta) \in]0;1 - CSL_{t\arg et}[$ a small change in β_i. Then γ_i can be approximated by:

(5) $\gamma_i = \lambda_i \cdot \Delta_i(\beta) + \varepsilon_i$, with $\varepsilon_i \in \Re$ representing the approximation error

Be θ_i the residual (decimal part) of the division of γ_i by λ_i. Then $\varepsilon_i = \theta_i \cdot \Delta_i(\beta)$.

$\Delta_i(\beta)$ determines the level of accuracy of the approximation, as the approximation error tends toward zero when $\Delta_i(\beta)$ tends towards 0.

(6) $$\lim_{\Delta_i(\beta)\to 0} \theta_i \cdot \Delta_i(\beta) = 0 \quad \Rightarrow \lim_{\Delta_i(\beta)\to 0} \varepsilon_i = 0$$

Be $\Delta_1(\beta) = \Delta_2(\beta) = \ldots = \Delta_p(\beta) = \Delta(\beta)$. Then $\Delta(\beta)$ determines the level of accuracy of the model, as the *smaller* $\Delta(\beta)$, the *smaller* the approximation errors ε_i will be, and the *closer* the solution will be to the optimum.

Assuming an infinitely high level of accuracy ($\Delta(\beta) \to 0$), we can approximate the optimal probability $\beta_{i(opt)}$ by applying (5) and (6) in (4):

(7) $$\beta_{i(opt)} = CSL_{t\arg et} + \lambda_i \cdot \Delta(\beta),$$

$$\text{with } \forall i \in [1;p], \lambda_i \cdot \Delta(\beta) \in]g\left(\frac{-\hat{Y}_i}{SE_i}, df\right) - CSL_{t\arg et}; 1 - CSL_{t\arg et}[\qquad \textbf{(QED)}$$

Conclusion: In other words, we know that the optimal probability $\beta_{i(opt)}$ of each product is an integral number of small changes Δ(β) (→ 0) away from the target customer service level CSL_{target}.

Part c)

Because $\beta_{i(opt)}$, i є [1;p] represent the optimal probabilities for each product i, and at the optimum $CSL_{\exp} = CSL_{t\arg et}$, we have, using equation (2):

(8) $$\frac{1}{p} \cdot \sum_{i=1}^{p} \beta_{i(opt)} = CSL_{t\arg et}$$

With (7) in (8), we obtain:

(9) $$\frac{1}{p} \cdot \sum_{i=1}^{p} (CSL_{t\arg et} + \lambda_i \cdot \Delta(\beta)) = CSL_{t\arg et} \Rightarrow \Delta(\beta) \cdot \sum_{i=1}^{p} \lambda_i = 0 \Rightarrow \sum_{i=1}^{p} \lambda_i = 0$$

$$\text{with } \forall i \in [1;p], \lambda_i \cdot \Delta(\beta) \in]-CSL_{t\arg et}; 1 - CSL_{t\arg et}[\qquad \textbf{(QED)}$$

Simply said, equation (9) means that for each small *positive* ($\lambda_i = 1$) change $+\Delta(\beta)$ made for a product i є [1;p], a small *negative* ($\lambda_j = -1$) change $-\Delta(\beta)$ must be made for a product j є [1;p] – { i }, thus verifying $\lambda_i + \lambda_j = 0$.

Conclusion: When a solution is optimal, then the sum of the number λ_i, i є [1;p], of small changes $\Delta(\beta)$ for all products is equal to 0.

*****

Note:

a) If $\sum_{i=1}^{p} \lambda_i > 0$, then $\frac{1}{p} \cdot \sum_{i=1}^{p} (CSL_{target} + \lambda_i \cdot \Delta(\beta)) > CSL_{target}$, i.e. $CSL_{exp} > CSL_{target}$

b) If $\sum_{i=1}^{p} \lambda_i < 0$, then $\frac{1}{p} \cdot \sum_{i=1}^{p} (CSL_{target} + \lambda_i \cdot \Delta(\beta)) < CSL_{target}$, i.e. $CSL_{exp} < CSL_{target}$

Thus, if $\sum_{i=1}^{p} \lambda_i \neq 0$, the resulting solution is either sub-optimal (a) or infeasible (b).

*****

Appendix 3: Computation of the Costs/ Savings of Adding/ Removing one small change Δ(β) and Global Optimality Definition

Demonstration:

a) Derive the formulas for computing the costs and savings of adding and removing one small change $\Delta(\beta)$ for a given product i.

b) Demonstrate that global optimality is reached whenever the minimal cost of adding a small change to one product exceeds the maximal saving from removing a small change from another product.

Note:

df	degrees of freedom
p	number of products in the model
X_i	quantity of product i to be on stock on the beginning of the observed month
Y_i	demand forecast for product i
SE_i	standard error of the forecast for product i
c_i	material unit costs of product I
β_i	probability of not stocking out of material for product i
$\Delta(\beta)$	small change in β_i
λ_i	integral number of small changes added or removed from CSL_{target} for product i

Part a)

We defined the function $g(t_i, df)$[23] which returns the probability of stocking out α_i for a given value of t_i. Be $r(\alpha_i , df)$ the function[24] returning the t-value t_i corresponding to a stocking-out probability α_i for df degrees of freedom, such that:

$$\textbf{(3)} \qquad g \circ r = g(r(\alpha_i, df), df) = \alpha_i$$

And:

$$\textbf{(4)} \qquad r \circ g = r(g(t_i, df), df) = t_i$$

If β_i is the probability of not stocking out of material for product i before an operation (i.e. addition or removal of a small change $\Delta(\beta)$ from product i), then be β'_i the probability of not stocking out of material for i after this operation.

If the operation consists of adding a small change $\Delta(\beta)$ to product i, then:

$$\textbf{(5a)} \qquad \beta'_i = \beta_i + \Delta(\beta)$$

If the operation consists of removing a small change $\Delta(\beta)$ from product i, then:

$$\textbf{(5b)} \qquad \beta'_i = \beta_i - \Delta(\beta)$$

Also, be t'_i the t-value after the operation. Then we have, using function $r(\alpha_i , df)$:

$$\textbf{(6)} \qquad t'_i = r(1 - \beta'_i, df)$$

---

[23] This function corresponds to the TDIST function of MS Excel (TVERT in German).
[24] This function corresponds to the TINV function of MS Excel.

Be $\Delta_i(t)$ the difference between t'_i and t_i , i.e. the small change in the t-value t_i corresponding to the small change $\Delta(\beta)$ in probability β_i. Then:

(7) $$\Delta_i(t) = t'_i - t_i$$

Using (6) in (7), we obtain:

(8) $$\Delta_i(t) = r((1-\beta'_i), df) - t_i$$

Be $C_i(\Delta(\beta))$ the cost of adding a small change $\Delta(\beta)$ in β_i to a product i, and $S_i(\Delta(\beta))$ the saving from removing a small change $\Delta(\beta)$ in β_i from product i.
Be $C_i(\Delta(t))$ the cost of adding the corresponding small change $\Delta_i(t)$ in t_i to a product i, and $S_i(\Delta(t))$ the saving from removing the corresponding small change $\Delta_i(t)$ in t_i from product i.

Then:

(9a) $$C_i(\Delta(\beta)) = C_i(\Delta(t))$$

(9b) $$S_i(\Delta(\beta)) = S_i(\Delta(t))$$

We know that:

(10) $$t_i = \frac{X_i - \hat{Y}_i}{SE_i}$$

t_i is the t-value corresponding to the quantity X_i before an operation, and t'_i the t-value corresponding to the quantity X'_i after the operation.
Be $\Delta_i(X)$ the difference between X'_i and X_i , i.e. the small change in the quantity X_i corresponding to the small change $\Delta_i(t)$ in the t-value t_i.

(11) $$\Delta_i(X) = X'_i - X_i$$

Using (10) and (11) in (7), we obtain:

(12) $$\Delta_i(t) = \frac{X'_i - \hat{Y}_i}{SE_i} - \frac{X_i - \hat{Y}_i}{SE_i} = \frac{X'_i - X_i}{SE_i} = \frac{\Delta_i(X)}{SE_i}$$

Be $C_i(\Delta(X))$ the cost of adding $\Delta_i(X)$ units to a product i, and $S_i(\Delta(X))$ the saving from removing $\Delta_i(X)$ units of a product i. Then, c_i being the material unit costs of product i, we have:

(13a) $$C_i(\Delta(X)) = c_i \cdot \Delta_i(X)$$

(13b) $$S_i(\Delta(X)) = -c_i \cdot \Delta_i(X)$$

Thus, using (12), (13a) and (13b), we have:

(14a) $$C_i(\Delta(t)) = SE_i \cdot c_i \cdot \Delta_i(t)$$

(14b) $$S_i(\Delta(t)) = -SE_i \cdot c_i \cdot \Delta_i(t)$$

Conclusion:

With (9a) and (14a) (resp. (9b) and (14b)), and introducing (5a), (5b) and (8), we conclude:

(15a) $C_i(\Delta(\beta)) = SE_i \cdot c_i \cdot \left[r((1-(\beta_i + \Delta(\beta))), df) - t_i \right]$

(15b) $S_i(\Delta(\beta)) = -SE_i \cdot c_i \cdot \left[r((1-(\beta_i + \Delta(\beta))), df) - t_i \right]$

These are the formulas used by the algorithm to determine the costs of an additional small change and the savings from removing a small change after each operation.

Part b)

Definitions:

$C_i(\Delta(\beta))$ is the cost of adding one small change to a product i (i □ [1;p]). The minimal cost of adding a small change to any product is then:

$$\min\left[C(\Delta(\beta))\right] = \min\{C_1(\Delta(\beta)); C_2(\Delta(\beta)); ...; C_p(\Delta(\beta))\} = \min\{C_i(\Delta(\beta)), i \in [1;p]\}$$

$S_i(\Delta(\beta))$ is the saving of removing one small change from product i (i □ [1;p]). The maximal saving of removing a small change to any product is then:

$$\max\left[S(\Delta(\beta))\right] = \max\{S_1(\Delta(\beta)); S_2(\Delta(\beta)); ...; S_p(\Delta(\beta))\} = \max\{S_i(\Delta(\beta)), i \in [1;p]\}$$

Assumption (**A**):

Assume that $\min\left[C(\Delta(\beta))\right] > \max\left[S(\Delta(\beta))\right]$, with the two following constraints satisfied:

/1 $\sum_{i=1}^{p} \lambda_i = 0$ and **/2** $\forall i \in [1;p], \lambda_i \cdot \Delta(\beta) \in \left] g\left(\frac{-\hat{Y}_i}{SE_i}, df\right) - CSL_{t\arg et}; 1 - CSL_{t\arg et}\right[$

Decision Rule:

Be TC the total inventory cost when assumption (**A**) is satisfied. If there do not exist any integers λ_i^* , i □ [1;p] such that the corresponding cost TC^* be smaller than TC and the two model constraints be satisfied, then assumption (**A**) is the optimality condition.

Demonstration:

Be i a product of the model (i □ [1;p]), then $S_i(\Delta(\beta))$ is the saving obtained by removing one small change $\Delta(\beta)$ from the current probability of not stocking out, known as β_i.

In Appendix 1, it has already been demonstrated that the only way to reduce costs is to remove a small change. If we remove one small change from product i, such that

$\lambda_{i(t+1)} \cdot \Delta(\beta) > g\left(\frac{-\hat{Y}_i}{SE_i}, df\right) - CSL_{t\arg et}$ (see constraint /2\\)

then the total inventory cost TC^* would indeed be lower than the cost corresponding to the initial assumption:

$TC^* = TC - S_i(\Delta(\beta)) \Rightarrow TC^* < TC$,

but the new state is not a feasible model solution, as constraint /1\\ is not satisfied:

$\lambda_{i(t+1)} = \lambda_{i(t)} - 1 \Rightarrow \sum_{i=1}^{p} \lambda_{i(t+1)} = \sum_{i=1}^{p} \lambda_{i(t)} - 1$

with assumption **(A)**: $\Rightarrow \sum_{i=1}^{p} \lambda_{i(t)} < 0 \Rightarrow CSL_{\exp} < CSL_{t\arg et}$ (t □ Z being the operation[25] index)
Thus, we would have to add one small change to another product *j* (indeed, adding a small change back to *i* would make us return to the initial situation) such that

$\lambda_{j(t)} \cdot \Delta(\beta) < 1 - CSL_{t\arg et}$ (see constraint /2\)

in order to satisfy all constraints again:

$$\lambda_{i(t+2)} = \lambda_{i(t+1)} + 1 \Rightarrow \sum_{i=1}^{p} \lambda_{i(t+2)} = \sum_{i=1}^{p} \lambda_{i(t+1)} + 1 = \sum_{i=1}^{p} \lambda_{i(t)} = 0 \Rightarrow CSL_{\exp} = CSL_{t\arg et}$$

However, with assumption **(A)**, for any product *j* (*j* □ [1;*p*]), the cost for one more small change noted $C_j(\Delta(\beta)$ is higher than the saving we just made on product *i*, noted $S_i(\Delta(\beta)$, as:

$$\min[C(\Delta(\beta)] > \max[S(\Delta(\beta)]$$

Thus: $C_j(\Delta(\beta) > S_i(\Delta(\beta) \Rightarrow C_j(\Delta(\beta) - S_i(\Delta(\beta) > 0$

From that, we deduce: $TC^* = TC - S_i(\Delta(\beta)) + C_j(\Delta(\beta) \Rightarrow TC^* > TC$ **(QED)**

It is consequently impossible to reduce inventory costs further if **(A)** is satisfied.

Conclusion:

Global optimality is reached when the *minimal* cost of *adding* a small change Δ(*β*) for a product *i* є [1;*p*] is higher than the *maximal* saving of *removing* a small change Δ(*β*) for a product *j* є [1;*p*] – { *i* }, provided both model constraints are satisfied:

- $\sum_{i=1}^{p} \lambda_i = 0 \Rightarrow CSL_{\exp} = CSL_{t\arg et} \Rightarrow CSL_{\exp} \geq CSL_{t\arg et}$
- $\forall i \in [1;p], \lambda_i \cdot \Delta(\beta) \in]g\left(\frac{-\hat{Y}_i}{SE_i}, df\right) - CSL_{t\arg et}; 1 - CSL_{t\arg et}[\quad \Rightarrow X_i \geq 0$

---

[25] An operation is the addition or removal of one small change Δ(*β*) from a product.

Appendix 4: Integrality Optimization

Demonstration:

a) Explain the two possibilities for converting decimal quantities to integers and derive the constraint on the expected CSL after the integrality optimization as compared to the expected CSL before the integrality optimization.

b) Derive the canonical integrality optimization model definition that the algorithm must solve after having determined the optimal decimal quantities.

Part a)

In Appendix 1 we demonstrated that the global optimum allowing decimal quantities satisfies the constraint $CSL_{\exp} = CSL_{target}$. The integrality optimization starts from that point and either truncates or rounds the optimal decimal quantities $X_{i(opt)}$, $i \in [1;p]$, $X_{i(opt)} \in \mathbb{R}^+$, thus affecting $CSL_{\exp}$.

Be $X^*_{i(opt)}$, $i \in [1;p]$, $X^*_{i(opt)} \in \mathbb{N}$, the optimal quantity after the integrality optimization, and be δ_i the decimal part of $X_{i(opt)}$, such that:

(1.) $\delta_i = X_{i(opt)} - Trunc\ (X_{i(opt)}, 0)$,

where $Trunc$ is the function that truncates $X_{i(opt)}$ to its integral part, and $\delta_i \in [0;1]$

Then, to convert $X_{i(opt)}$ into an integer, one can either remove δ_i from $X_{i(opt)}$ (i.e. truncate), or add $1 - \delta_i$ to $X_{i(opt)}$ (i.e. round up to the higher integer).

Thus, the optimal integral quantity for a product i is either one of the following:

(2a.) $X^*_{i(opt)} = X_{i(opt)} - \delta_i$ (Truncating)

(2b.) $X^*_{i(opt)} = X_{i(opt)} + 1 - \delta_i$ (Rounding)

Be $t^*_{i(opt)}$ the optimal t-value after the integrality optimization.
Then, using the known t-value formula, we derive from (2a) and (2b):

(3a.) $$t^*_{i(opt)} = \frac{X_{i(opt)} - \delta_i - \hat{Y}_i}{SE_i} \quad \text{(Truncating)}$$

(3b.) $$t^*_{i(opt)} = \frac{X_{i(opt)} + 1 - \delta_i - \hat{Y}_i}{SE_i} \quad \text{(Rounding)}$$

Be $\beta^*_{i(opt)}$ the optimal probability of delivering the demand for product i on time after the integrality optimization.

Recalling that $t_{i(opt)} = \dfrac{X_{i(opt)} - \hat{Y}_i}{SE_i}$ and $\beta_{i(opt)} = g(t_{i(opt)}, df)$, we see that:

(4a.) When truncating: $t^*_{i(opt)} \le t_{i(opt)} \Rightarrow \beta^*_{i(opt)} \le \beta_{i(opt)}$

(4b.) When rounding: $t^*_{i(opt)} \ge t_{i(opt)} \Rightarrow \beta^*_{i(opt)} \ge \beta_{i(opt)}$

Be φ_i the gain/loss in probability from rounding/truncating the optimal decimal quantity.

(5.) $\varphi_i = \beta^*_{i(opt)} - \beta_{i(opt)}$

We know that:

(6.) $$CSL_{\exp} = \frac{1}{p} \cdot \sum_{i=1}^{p} \beta_i$$

Be $CSL^*_{\exp}$ the expected level of CS after the integrality optimization.

Although it might not be possible to respect the global optimality condition $CSL_{\exp} = CSL_{target}$ fully when converting decimal quantities into integers, the deviation must remain such that:

(7.) $$CSL^*_{\exp} \geq CSL_{\exp} \Rightarrow CSL^*_{\exp} \geq CSL_{target}$$

This is important for respecting the model constraint.

Part b)

We know that reducing the quantity on stock reduces the costs and vice versa.

Thus, each product i has a potential cost $C_i(\delta_i)$ of rounding its optimal quantity up to the next integer and a potential saving $S_i(1 - \delta_i)$ of truncating its optimal quantity down to its integral part. At the same time, each product may cause a gain in $CSL_{\exp}$ of φ_i / p for rounding or a loss in $CSL_{\exp}$ of - φ_i / p for truncating.

Equation (7) implies, using (6):

(8.) $$\frac{1}{p} \cdot \sum_{i=1}^{p} \beta^*_{i(opt)} \geq \frac{1}{p} \cdot \sum_{i=1}^{p} \beta_{i(opt)} \Leftrightarrow \frac{1}{p} \cdot \sum_{i=1}^{p} g(t^*_{i(opt)}, df) \geq \frac{1}{p} \cdot \sum_{i=1}^{p} g(t_{i(opt)}, df)$$

Be p_t the number of products to be truncated and p_r the number of products to be rounded, such that $p_t + p_r = p$.
Be i_t є $[1; p_t]$ the products to be truncated and i_r є $[1; p_r]$ the products to be rounded.

Using (3a) and (3b), the $CSL^*_{\exp}$ equals:
(9.)

$$\frac{1}{p} \cdot \sum_{i=1}^{p} g(t^*_{i(opt)}, df) = \frac{1}{p_t} \cdot \sum_{i_t=1}^{p_t} g\left(\frac{X_{i_t(opt)} - \delta_{i_t} - \hat{Y}_{i_t}}{SE_{i_t}}, df\right) + \frac{1}{p_r} \cdot \sum_{i_r=1}^{p_r} g\left(\frac{X_{i_r(opt)} + 1 - \delta_{i_r} - \hat{Y}_{i_r}}{SE_{i_r}}, df\right)$$

From equations (14a) and (14b) of Appendix 3, we derive:

(9a.) $$C_i(\delta_i) = SE_i \cdot c_i \cdot \delta_i$$

(9b.) $$S_i(1 - \delta_i) = -SE_i \cdot c_i \cdot (1 - \delta_i)$$

Then integrality optimization model is thus defined by:

(10.) $$\sum_{i_t=1}^{p_t} S_{i_t}(1 - \delta_{i_t}) + \sum_{i_r=1}^{p_r} C_{i_r}(\delta_{i_r}) \overset{!}{=} \min$$

$$s.d.\ \frac{1}{p_t} \cdot \sum_{i_t=1}^{p_t} g\left(\frac{X_{i_t(opt)} - \delta_{i_t} - \hat{Y}_{i_t}}{SE_{i_t}}, df\right) + \frac{1}{p_r} \cdot \sum_{i_r=1}^{p_r} g\left(\frac{X_{i_r(opt)} + 1 - \delta_{i_r} - \hat{Y}_{i_r}}{SE_{i_r}}, df\right) \geq 0$$

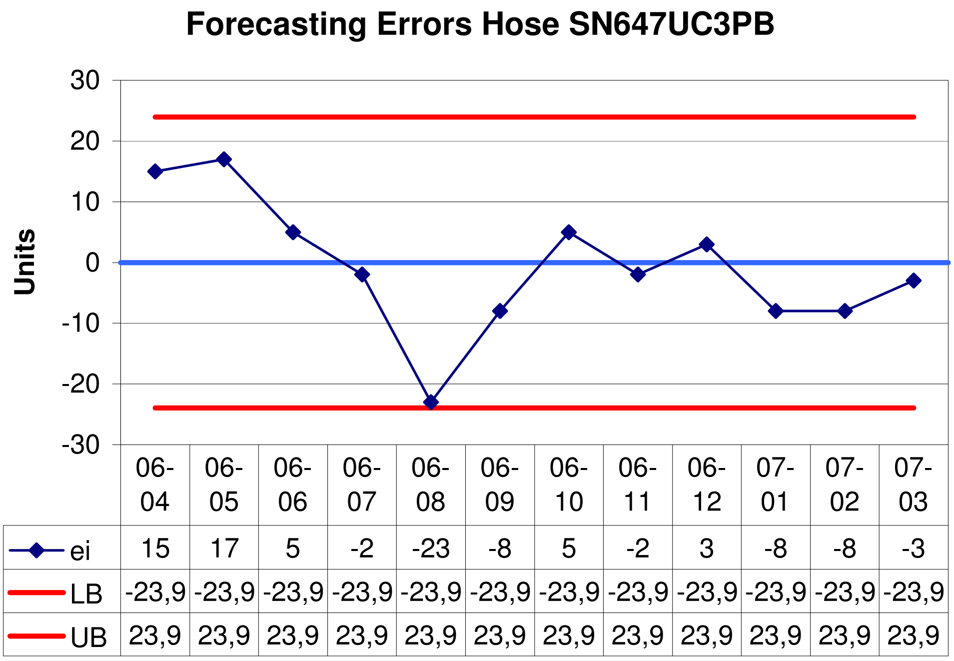
This model is solved by the algorithm after the decimal global optimum has been computed as described in Appendices 1 to 3.

Appendix 5: Forecasting Bias Tests

Panel A: Tracking Signal (Confidence Interval)

Example of the hose SN647UC3PB

The demand forecasting errors e_i over the period April 2006 to March 2007 using the four-month moving average methods were the following:



	06-04	06-05	06-06	06-07	06-08	06-09	06-10	06-11	06-12	07-01	07-02	07-03
ei	15	17	5	-2	-23	-8	5	-2	3	-8	-8	-3
LB	-23,9	-23,9	-23,9	-23,9	-23,9	-23,9	-23,9	-23,9	-23,9	-23,9	-23,9	-23,9
UB	23,9	23,9	23,9	23,9	23,9	23,9	23,9	23,9	23,9	23,9	23,9	23,9

From those data, we compute both the mean error and standard deviation:

(1) $ME = \frac{1}{n} \cdot \sum_{i=1}^{n} e_i$, here: $ME = -0.75$

(2) $s = \sqrt{\frac{\sum_{i=1}^{n} (e_i - ME)^2}{n-1}}$, here: $s = 10.88$

Bias shall be determined at a confidence level of 95%. As the standard deviation used comes from a small sample of twelve data points and the population standard deviation is unknown, a t-distribution with eleven degrees of freedom is used rather than a normal distribution. Thus:

(3) $\alpha = 0.05$, and $t_{crit} = t_{(\alpha=0.05;df=11)} \approx \pm 2.2$

The program is using a tracking signal based on the confidence interval built around the assumed population mean $\mu_{ME} = 0$. The confidence interval is:

(4) CI: $0 \pm t_{crit} \cdot s$, here: CI: [-2.2 · 10.88; 2.2 · 10.88] i.e. CI: [-23.95; 23.95]

As soon as one forecasting error falls out of this tracking range (see Graph), the user is alerted that there might be evidence of bias. The bias-index depends on the number of errors out of bound (in this example, the index is 0), but also on the result of the binomial test presented next.

Panel B: Binomial Test

If the assumption of normally distributed forecasting errors is valid, there must be the same number of positive and negative deviations over time. In this binomial test, the twelve last forecasting errors are considered to estimate if there is evidence of bias. The trials can be assumed independent based on the autocorrelation analysis performed in Appendix 9. Let a success be to obtain a positive forecasting error.

Number of Trials (n)	12	(over last twelve months)
Probability of success (pi)	50%	(normal distribution)

Number of successes (X)	**0**	**1**	**2**	**3**	**4**	**5**	**6**
P(X)	0,02%	0,29%	1,61%	5,37%	12,08%	19,34%	22,56%
Cum. P(X) ascendant	0,02%	0,32%	1,93%	7,30%	19,38%	38,72%	61,28%
Cum. P(X) descendant	61,28%	61,25%	60,96%	59,35%	53,98%	41,89%	22,56%

Number of successes (X)	**7**	**8**	**9**	**10**	**11**	**12**
P(X)	19,34%	12,08%	5,37%	1,61%	0,29%	0,02%
Cum. P(X) ascendant	61,28%	61,28%	61,28%	61,28%	61,28%	61,28%
Cum. P(X) descendant	38,72%	19,38%	7,30%	1,93%	0,32%	0,02%

Significance Level:	3,86%	P(X=0)+P(X=1)+P(X=2)+P(X=10)+P(X=11)+P(X=12)
Success Interval:	3-9	(number of successes in over 96% of cases)

At a level of significance of about 4%, there is a 96% probability of obtaining a number of positive (resp. negative) deviations between 3 and 9 over the past twelve months. For the bias evaluation in the presented model, it was thus defined that there was evidence for bias when the number of positive (resp. negative) deviations over the past twelve months falls out of this range, i.e. equals 0, 1, 2, 10, 11, or 12.

Recalling the example of Panel A, there are five positive deviations and seven negative deviations, which lies within the confidence area. Thus, there is no evidence of bias over the past twelve month according to this binomial test.

Appendix 6: Examples of Forecast Computations

This Appendix shows a forecast computation example for hose 5Q05B6ER7D with all seven techniques initially assessed for there accuracy and bias. The summarized results for all products are provided in Appendix 7.

Panel A: Standard Decomposition

Period	Y (t)	QS	Yp (m)	e (t)
06-07	500	4,20%	165	335
06-08	0	25,21%	988	-988
06-09	2.000	70,59%	2.767	-767
06-10	2.000	59,30%	2.466	-466
06-11	3.000	11,63%	484	2.516
06-12	2.000	29,07%	1.209	791
07-01	1.800	52,17%	2.849	-1.049
07-02	2.000	10,87%	593	1.407
07-03	4.450	36,96%	2.018	2.432
07-04	1.000	30,77%	1.861	-861
07-05	2.900	21,98%	1.329	1.571
07-06	500	47,25%	2.858	-2.358

Quarterly Forecasts:

Q (q=9)	3920
Q (q=10)	4159
Q (q=11)	5460
Q (q=12)	6047

MAD:	**1.295**
ME:	**214**

%QuarterlySales (QS):

01	52,17%
02	10,87%
03	36,96%
04	30,77%
05	21,98%
06	47,25%
07	4,20%
08	25,21%
09	70,59%
10	59,30%
11	11,63%
12	29,07%

Trend Coefficients:

Trend	354
Base	1146

Seasonal Factors:

SF (q=9)	0,91
SF (q=10)	0,89
SF (q=11)	1,08
SF (q=12)	1,12

Parameters optimized over the period 04-07 to 06-06

The forecast for month m is computed as follows:

$$Yp_m = \hat{Y}_m = QS \cdot \left(SF_q \cdot \left(Base + q \cdot Trend\right)\right)$$

e.g.: $Yp_{06-07} = \hat{Y}_{06-07} = 4.20\% \cdot \left(0.91 \cdot \left(1145.5 + 9 \cdot 353.7\right)\right) = 165$

where QS is the percentage of quarterly sales falling in a given month over the past two years
SF is the seasonal factor of quarter q
$Base$ is the intercept of the regression line
$Trend$ is the coefficient of the regression line

Panel B: Exponential Smoothing

Period	Y (t)	Yp (t+3)	e (t)
06-04	3700	550	
06-05	1.000	839	
06-06	1.400	1791	
06-07	500	1775	-1275
06-08	0	902	-902
06-09	2.000	1639	361
06-10	2.000	1279	721
06-11	3.000	551	2449
06-12	2.000	1779	221
07-01	1.800	1560	240
07-02	2.000	1504	496
07-03	4.450	1865	2585
07-04	1.000	1653	-653
07-05	2.900	1697	1203
07-06	500	2871	-2371

MAD	**1123**
ME	**256**

LFC(t)	549,75
LFC(t+1)	839,25
LFC(t+2)	1790,70
Alpha	0,39

Parameters from prior optimization over the period 05-06 to 06-05.

The forecast for month t+3 is computed as follows:

$$Yp_{t+3} = \hat{Y}_{t+3} = \alpha Y_t + (1-\alpha)\hat{Y}_t$$

e.g. $Yp_{06-07} = \hat{Y}_{06-07} = \alpha Y_{06-04} + (1-\alpha)\hat{Y}_{06-04} = 0.389 \cdot 3{,}700 + 0.611 \cdot 550 = 1{,}775$

Panel C: Trend Adjusted Exponential Smoothing (Holt's Method)

Period	Y (t)	A (t)	T (t)	Yp (t+3)	e (t)
06-04	3700	1087	59		
06-05	1000	1144	142		
06-06	1400	1288	165		
06-07	500	1434	175	1265	-765
06-08	0	1578	152	1571	-1571
06-09	2.000	1735	153	1783	217
06-10	2.000	1890	199	1961	39
06-11	3.000	2108	260	2033	967
06-12	2.000	2360	317	2195	-195
07-01	1.800	2660	340	2488	-688
07-02	2.000	2979	355	2887	-887
07-03	4.450	3356	415	3311	1139
07-04	1.000	3716	457	3679	-2679
07-05	2.900	4148	445	4044	-1144
07-06	500	4510	423	4603	-4103

Initial Values

A(initial)	1087,35
T(initial)	59,30

Coefficients

Alpha:	0,02
Beta:	0,98

Parameters from prior optimization over the period 05-06 to 06-05.

MAD	**1200**
ME	**-806**

The forecast for month t+3 is computed as follows (Hanke / Reitsch, 1995):

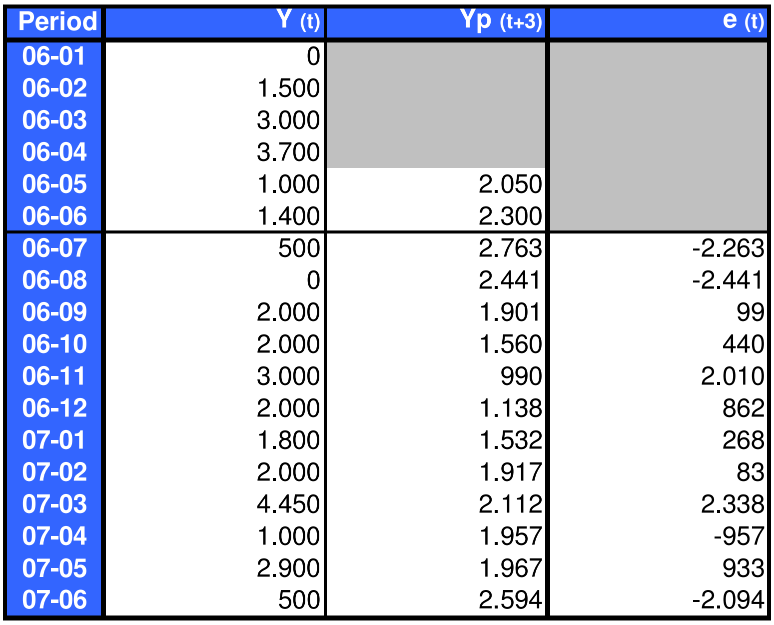
Intercept: $A_t = \alpha Y_t + (1-\alpha)\cdot(A_{t-1} + T_{t-1})$

Coefficient: $T_t = \beta\cdot(A_t - A_{t-1}) + (1-\beta)\cdot T_{t-1}$

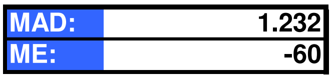
Then: $Yp_{t+3} = \hat{Y}_{t+3} = A_t + 3\cdot T_t$, with $\hat{Y}_{t+3} \geq 0$

e.g. $Yp_{06-07} = \hat{Y}_{06-07} = A_{06-04} + 3\cdot T_{06-04} = 1{,}087 + 3\cdot 59.3 = 1{,}265$

Panel D: Four-Month Moving Average



Period	Y (t)	Yp (t+3)	e (t)
06-01	0		
06-02	1.500		
06-03	3.000		
06-04	3.700		
06-05	1.000	2.050	
06-06	1.400	2.300	
06-07	500	2.763	-2.263
06-08	0	2.441	-2.441
06-09	2.000	1.901	99
06-10	2.000	1.560	440
06-11	3.000	990	2.010
06-12	2.000	1.138	862
07-01	1.800	1.532	268
07-02	2.000	1.917	83
07-03	4.450	2.112	2.338
07-04	1.000	1.957	-957
07-05	2.900	1.967	933
07-06	500	2.594	-2.094



MAD:	1.232
ME:	-60

The forecast for month t+3 is computed as follows:

$$Yp_{t+3} = \hat{Y}_{t+3} = \frac{Y_{t-1} + Y_t + \hat{Y}_{t+1} + \hat{Y}_{t+2}}{4}$$

e.g.

$$Yp_{06-07} = \hat{Y}_{06-07} = \frac{Y_{06-03} + Y_{06-04} + \hat{Y}_{06-05} + \hat{Y}_{06-06}}{4} = \frac{3{,}000 + 3{,}700 + 2{,}050 + 2{,}300}{4} = 2{,}763$$

Panel E: Double Four-Month Moving Average

Period	Y (t)	M (t)	M' (t)	a (t)	b (t)	Yp (t+3)	e (t)
05-10	900						
05-11	1.000						
05-12	1.000						
06-01	0	725					
06-02	1.500	875					
06-03	3.000	1.375					
06-04	3.700	2.050	1.256	2.844	529		
06-05	1.000	2.300	1.650	2.950	433		
06-06	1.400	2.275	2.000	2.550	183		
06-07	500	1.650	2.069	1.231	-279	4.431	-3.931
06-08	0	725	1.738	-288	-675	4.250	-4.250
06-09	2.000	975	1.406	544	-288	3.100	-1.100
06-10	2.000	1.125	1.119	1.131	4	394	1.606
06-11	3.000	1.750	1.144	2.356	404	0	3.000
06-12	2.000	2.250	1.525	2.975	483	0	2.000
07-01	1.800	2.200	1.831	2.569	246	1.144	656
07-02	2.000	2.200	2.100	2.300	67	3.569	-1.569
07-03	4.450	2.563	2.303	2.822	173	4.425	25
07-04	1.000	2.313	2.319	2.306	-4	3.306	-2.306
07-05	2.900	2.588	2.416	2.759	115	2.500	400
07-06	500	2.213	2.419	2.006	-138	3.341	-2.841

MAD:	**1974**
ME:	**-693**

The forecast for month $t+3$ is computed as follows (Hanke / Reitsch, 1995):

Simple Moving Average: $M_t = \dfrac{Y_t + Y_{t-1} + Y_{t-2} + Y_{t-3}}{4}$

Double Moving Average: $M'_t = \dfrac{M_t + M_{t-1} + M_{t-2} + M_{t-3}}{4}$

Intercept: $A_t = 2 \cdot M_t - M'_t$

Coefficient: $B_t = \dfrac{2}{4-1} \cdot (M_t - M'_t)$

Then: $Yp_{t+3} = \hat{Y}_{t+3} = A_t + 3 \cdot B_t$, with $\hat{Y}_{t+3} \geq 0$

Example:

$$M_{06-04} = \frac{Y_{06-04} + Y_{06-03} + Y_{06-03} + Y_{06-02}}{4} = \frac{3{,}700 + 3{,}000 + 1{,}500 + 0}{4} = 2{,}050$$

$$M'_{06-04} = \frac{M_{06-04} + M_{06-03} + M_{06-03} + M_{06-02}}{4} = \frac{2{,}050 + 1{,}375 + 875 + 725}{4} = 1{,}256$$

$$A_{06-04} = 2 \cdot M_{06-04} - M'_{06-04} = 2 \cdot 2{,}050 - 1{,}256 = 2{,}844$$

$$B_{06-04} = \frac{2}{4-1} \cdot (M_{06-04} - M'_{06-04}) = \frac{2}{3} \cdot (2{,}050 - 1{,}256) = 529$$

Hence: $Yp_{06-07} = \hat{Y}_{06-07} = A_{06-04} + 3 \cdot B_{06-04} = 2{,}844 + 3 \cdot 529 = 4{,}431$

Panel F: Trend/Seasonality Adjusted Naïve Method

Period	Y (t)	Yp (t+3)	e (t)
06-01	0		
06-02	1.500		
06-03	3.000		
06-04	3.700		
06-05	1.000		
06-06	1.400	1.750	
06-07	500	3.063	-2.563
06-08	0	3.716	-3.716
06-09	2.000	1.004	996
06-10	2.000	1.401	599
06-11	3.000	500	2.500
06-12	2.000	0	2.000
07-01	1.800	2.000	-200
07-02	2.000	2.000	0
07-03	4.450	3.000	1.450
07-04	1.000	2.000	-1.000
07-05	2.900	1.800	1.100
07-06	500	2.000	-1.500

MAD:	**1.469**
ME:	**-28**

The forecast for month t+3 is computed as follows:

$$Yp_{t+3} = \hat{Y}_{t+3} = Y_{t-1} + \frac{(\hat{Y}_{t+2} - Y_{t-2})}{4}$$

e.g. $Yp_{06-07} = \hat{Y}_{06-07} = Y_{06-03} + \frac{(\hat{Y}_{06-06} - Y_{06-02})}{4} = 3{,}000 + \frac{(1{,}750 - 1{,}500)}{4} = 3{,}063$

Panel G: Naïve Method

Period	Y (t)	Yp (t+3)	e (t)
06-04	3700		
06-05	1000		
06-06	1400		
06-07	500	3700	-3200
06-08	0	1000	-1000
06-09	2000	1400	600
06-10	2000	500	1500
06-11	3000	0	3000
06-12	2000	2000	0
07-01	1800	2000	-200
07-02	2000	3000	-1000
07-03	4450	2000	2450
07-04	1000	1800	-800
07-05	2900	2000	900
07-06	500	4450	-3950

MAD	**1550**
ME	**-142**

The forecast for month t+3 is computed as follows:

$$Yp_{t+3} = \hat{Y}_{t+3} = Y_{t}$$

e.g. $Yp_{06-07} = \hat{Y}_{06-07} = Y_{06-04} = 3{,}700$

Appendix 7: Forecasting Techniques Evaluation Results

Panel A: Accuracy measured by MAD

This table shows the MAD for all 98 products selected for the model for the first six months of the evaluation period (see Appendix 12) for the seven forecasting techniques presented in Appendix 6 (Panel A to G), from July 06 to June 07. The example computations in Appendix 6 correspond to the green highlighted product.

Item	DataPoints	A	B	C	D	E	F	G	BestFC
5Q05B6ER7D	**24**	**1.295,08**	**1.123,13**	**1.199,25**	**1.232,17**	**1.973,67**	**1.468,58**	**1.550,00**	**B**
4DDY011F67	24	1.069,17	1.084,40	2.243,83	700,01	1.611,00	1.253,80	886,00	D
3K5691Q8LX	24	134,17	149,64	999,67	187,22	238,92	234,67	202,08	A
8W2A45C39J	24	592,33	591,26	461,25	444,24	439,42	622,41	615,92	E
0U1F67A5CH	24	562,33	665,57	1.072,00	714,56	875,75	902,15	680,92	A
2Y7KKF314L	24	18,50	34,57	48,00	19,24	20,00	19,75	13,83	G
4367H52631	24	1.010,83	1.130,13	926,00	920,50	1.058,00	949,17	1.457,58	D
3B4367H526	24	675,67	490,97	985,42	730,75	1.085,33	1.193,00	675,00	B
X7D20X8B2	24	318,75	263,54	214,75	219,11	267,00	275,00	316,67	C
PBRXV4Z886	24	63,08	60,17	481,33	70,80	82,58	115,00	57,50	G
H589G74N5Y	24	1.035,33	616,82	830,17	645,76	963,67	879,84	634,58	B
9H05K228ZV	24	1.711,25	1.592,91	1.693,33	1.593,58	2.280,83	2.013,37	2.018,67	B
YKB9N331CX	24	152,75	114,87	125,42	107,98	128,83	158,80	135,42	D
0TTH647UD3	17		401,42	436,75	281,59	334,00	219,61	401,42	F
MH3D5691J8	24	1.415,83	1.370,40	1.290,00	1.301,34	1.493,67	1.167,00	1.661,42	F
UY3SJOMZ0L	20		48,08	100,75	41,99	47,75	56,97	48,08	D
AE6Y688F2R	24	499,33	249,92	720,92	388,29	359,00	371,88	249,92	B
7KJEF3LTX	18		292,19	313,00	287,26	517,08	434,55	408,33	D
COF8R2203B	24	103,75	59,82	118,58	80,96	96,67	145,70	41,67	G
6S9MY0L5F	24	276,17	246,42	225,08	188,23	299,67	252,66	246,42	D
7U1Y35A39H	24	361,67	234,79	460,92	327,08	614,08	453,12	349,92	B
H5COF8R220	24	2.266,50	1.769,03	2.141,67	1.497,93	2.552,75	1.803,55	2.047,08	D
VZ4UD7HAS5	24	2,25	1,74	2,42	2,05	3,25	2,82	2,67	B
Z1KO2J052P	24	33,00	24,12	44,00	26,75	39,25	30,81	26,42	B
3Z08U7XBJN	24	17,42	10,26	22,25	10,57	18,50	13,38	14,67	B
9S1957Y4UG	24	9,67	10,33	16,42	5,60	10,42	7,04	10,33	D
8GW4ZM50YT	24	28,58	21,13	34,25	25,51	37,08	31,27	34,50	B
DZL16N439	24	10,33	7,66	25,00	8,10	14,42	11,25	13,92	B
9H2C3NJRV	24	36,33	31,42	59,42	25,58	29,33	25,98	25,08	G
60752XWRR6	24	22,08	15,16	26,67	15,45	17,33	18,65	15,50	B
QCT3E875TP	24	19,67	18,17	21,17	16,67	31,00	22,51	25,42	D
X845V37C86	24	22,58	16,79	20,17	15,04	20,00	16,77	18,75	D
2ZH589F74U	24	8,50	6,15	12,25	5,39	8,17	10,02	5,83	D
16EJH8S331	24	35,17	27,18	57,50	27,36	25,00	25,00	26,83	E
4W6DD3GK	24	22,00	20,67	25,08	25,25	40,33	35,07	36,42	B
SN647UC3PB	24	9,50	8,11	23,58	9,33	19,25	12,81	13,75	B
ZU192BQUDO	24	12,83	12,27	14,25	12,10	10,83	12,87	10,33	G
AE6Y688F2R	24	35,25	37,72	47,25	39,29	61,42	50,46	53,17	A
CYG47VE73L	24	26,92	22,33	47,42	23,33	31,33	24,51	18,33	G
GK8EV3YLJ9	18		12,24	15,58	11,51	18,75	14,00	13,33	D
RM566SBFUZ	24	11,25	6,74	18,58	8,11	17,17	12,27	13,00	B
088W9145C3	24	13,58	8,61	15,33	7,38	13,50	10,52	11,67	D
314F5479L7	24	17,00	12,10	47,00	13,21	13,17	11,35	10,75	G
5HGB102QU	17		22,26	63,83	17,95	34,42	20,41	20,00	D
488HGB2L	24	91,83	60,59	97,33	61,46	81,00	76,27	65,33	B
21H257G516	24	22,75	18,98	71,92	21,49	39,33	32,88	31,08	B
SW5QH8KX97	24	193,25	229,60	183,67	182,17	247,08	109,58	278,00	F
S69A7950PC	24	396,25	416,68	415,92	325,23	409,42	335,72	294,17	G
1F6715CG8	24	14,25	10,33	7,75	8,45	5,08	8,50	5,33	E
75LOW81EU0	24	17,42	42,58	45,08	19,43	20,42	19,41	14,25	G
C36S1524GT	24	35,50	29,35	60,67	23,62	21,83	25,71	21,67	G
ZE16N119V0	24	35,08	28,90	85,08	25,81	46,58	32,64	35,58	D
0J16M008B9	24	10,25	8,03	17,83	9,26	16,75	13,44	12,58	B
FQAMDXP442	24	13,67	11,07	13,92	9,55	11,92	8,91	15,58	F
0923X18E94	24	20,25	16,75	25,83	16,91	18,33	13,31	28,25	F
EAJN2H041V	24	9,42	6,51	16,42	7,66	15,00	10,61	12,17	B
4992D3NJ	24	474,25	845,79	1.094,67	429,67	335,67	388,88	291,67	G
F47U263749	24	1.145,00	930,04	2.080,17	880,67	634,83	680,02	526,83	G
BW903DL7FK	24	294,75	233,40	332,67	277,11	327,33	366,01	216,50	G
57580O8JV	24	285,17	184,98	260,67	209,90	326,58	258,00	205,17	B
RMLG314NVZ	24	550,92	371,86	357,00	357,22	497,08	416,16	350,92	G
93UUPW7ZW	24	102,92	77,23	82,83	69,89	95,00	86,35	70,00	D
4UK7NA0MMH	24	8,50	8,81	12,17	8,88	10,75	9,31	13,00	A
PD5G6RZ2MY	24	2.664,50	2.449,13	2.766,17	2.547,80	2.991,00	2.658,33	3.283,33	B
Y8K220ZVC3	24	1.216,42	1.225,64	2.477,83	1.457,58	2.006,67	1.526,76	1.758,33	A
FAA22G6T46	24	64,17	51,81	111,17	46,66	56,75	72,08	78,33	D
7KJEE3LTX	17		24,17	43,33	37,88	40,25	61,01	24,17	B
6S1524GT5F	24	115,25	104,84	221,75	116,71	146,42	127,26	89,58	G
387FEZ314G	24	81,08	38,84	95,42	45,03	74,83	63,18	37,50	G
5F34580O8	24	28,33	27,78	67,17	22,97	35,92	28,92	31,50	D
KO2052P6B	24	31,92	25,17	52,33	22,57	24,00	22,16	24,08	F

The right column indicates the most accurate method for each product.

Panel B: Overview of Accuracy Ranking

Number of products assessed: 98

	Rank						
Method	**1**	**2**	**3**	**4**	**5**	**6**	**7**
SD	6	3	20	11	13	24	21
ES	29	29	14	11	7	6	2
HM	1	5	7	6	12	18	49
MA	27	32	22	14	3	0	0
DMA	5	9	6	12	19	34	13
NTM	12	6	20	24	25	6	5
NM	18	14	9	20	19	10	8

Notes:

SD: Seasonal Decomposition Method
ES: Exponential Smoothing
HM: Holt's Exponential Smoothing Method
MA: Four-Month Moving Average
DMA: Four-Month Double Moving Average
NTM: Naive Method with Trend & Seasonality
NM: Naive Method

From the accuracy ranking, three methods can be clearly identified that outperform the others: ES, MA and NM. For 74 out of 98 products, one of the three methods is the best. It has also be determined that there is, for each of the 98 products, at least one of the three methods among the three best performing.

Panel C: Forecasting Method Bias Assessment

This table shows that there is always at least one of the three best performing methods that delivers unbiased results when conducting the tests described in Appendix 5.

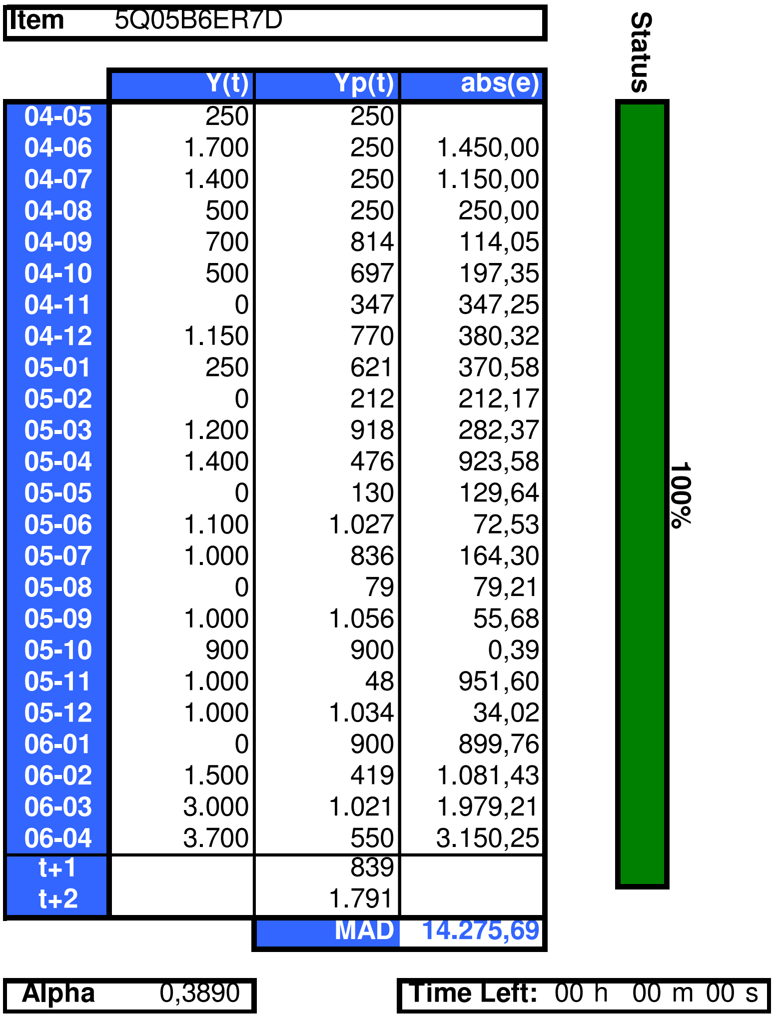
	Bias Index			
Method	**0**	**1**	**2**	**3**
SD	65	23	6	4
ES	67	7	16	8
HM	46	8	6	38
MA	56	6	27	9
DMA	73	22	3	0
NTM	76	19	2	1
NM	56	0	30	12

Note to Bias Index:

0 No evidence of bias
1 Slight evidence of bias
2 Medium evidence of bias
3 Strong evidence of bias

In the model, a forecasting method is considered to be biased when it has a bias-index of 3 based on the two bias tests.

Appendix 8: Smoothing Factor Optimization Algorithm



Item 5Q05B6ER7D

	Y(t)	Yp(t)	abs(e)
04-05	250	250	
04-06	1.700	250	1.450,00
04-07	1.400	250	1.150,00
04-08	500	250	250,00
04-09	700	814	114,05
04-10	500	697	197,35
04-11	0	347	347,25
04-12	1.150	770	380,32
05-01	250	621	370,58
05-02	0	212	212,17
05-03	1.200	918	282,37
05-04	1.400	476	923,58
05-05	0	130	129,64
05-06	1.100	1.027	72,53
05-07	1.000	836	164,30
05-08	0	79	79,21
05-09	1.000	1.056	55,68
05-10	900	900	0,39
05-11	1.000	48	951,60
05-12	1.000	1.034	34,02
06-01	0	900	899,76
06-02	1.500	419	1.081,43
06-03	3.000	1.021	1.979,21
06-04	3.700	550	3.150,25
t+1		839	
t+2		1.791	
		MAD	14.275,69

Status

100%

Alpha 0,3890

Time Left: 00 h 00 m 00 s

The alpha-optimization algorithm looks for the smoothing factor α such that, over the twenty-four months (May 04 to April 06) preceding the test period (July 06 to June 07), the mean average deviation is the smallest. Indeed, it must be reminded that the forecast for July 06 would have been done based on the demand data until April 06.

In the example of product 5Q05B6ER7D, this factor is 0.3890.

The optimization process takes about 4/5 seconds per product.

Appendix 9: Autocorrelation Analysis

This table shows the autocorrelation coefficient R_k for n period lags for each of the 98 products, as computed by:

$$R_k = \frac{\sum_{i=(t-n+1)+k}^{t} (Y_i - \overline{Y})(Y_{i-k} - \overline{Y})}{\sum_{i=t-n+1}^{t} (Y_i - \overline{Y})^2}$$

Item	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=11	n=12	Max	Lag
5Q05B6ER7D	0,29	0,27	0,07	0,07	0,10	0,24	0,07	0,07	0,07	-0,02	0,24	0,19	**29,05%**	**n=1**
4DDY011F67	0,38	0,13	0,22	0,11	0,14	0,20	-0,02	0,05	0,08	0,08	0,29	0,06	**37,63%**	**n=1**
3K5691Q8LX	0,27	-0,09	-0,08	-0,08	0,15	0,11	-0,20	-0,08	0,09	0,22	0,21	0,14	**26,67%**	**n=1**
8W2A45C39J	-0,09	-0,09	0,19	0,24	-0,10	0,19	-0,18	0,11	-0,12	-0,02	0,10	0,02	**24,02%**	**n=4**
0U1F67A5CH	0,14	0,19	0,20	0,16	-0,04	-0,06	0,01	0,08	-0,03	0,10	0,01	0,25	**24,96%**	**n=12**
2Y7KKF314L	0,40	0,49	0,30	0,18	0,30	0,08	0,12	0,02	0,04	-0,07	0,02	-0,15	**49,38%**	**n=2**
4367H52631	-0,31	0,29	-0,20	0,36	-0,06	0,22	-0,19	0,40	-0,25	0,15	-0,09	0,17	**40,34%**	**n=8**
3B4367H526	0,06	-0,08	0,29	-0,01	0,04	0,05	-0,05	0,08	0,15	-0,09	0,10	0,16	**28,81%**	**n=3**
X7D20X8B2	-0,08	-0,15	0,15	0,13	0,22	-0,16	0,06	0,28	-0,04	-0,21	0,14	0,12	**28,19%**	**n=8**
PBRXV4Z886	0,02	-0,02	0,12	-0,14	0,23	0,04	0,03	-0,09	-0,17	0,11	-0,07	-0,15	**22,96%**	**n=5**
H589G74N5Y	0,04	0,02	0,19	0,04	-0,03	-0,05	0,21	0,02	-0,12	0,16	0,20	-0,18	**20,70%**	**n=7**
9H05K228ZV	-0,20	0,18	0,04	-0,03	-0,07	0,02	-0,07	0,19	0,03	-0,04	0,03	0,06	**19,35%**	**n=8**
YKB9N331CX	0,13	0,04	0,23	0,05	0,05	0,31	-0,12	-0,01	0,16	-0,03	-0,03	0,18	**31,14%**	**n=6**
0TTH647UD3	0,01	0,54	-0,05	0,43	0,02	0,34	0,02	0,13	-0,09	0,25	-0,02	0,03	**53,68%**	**n=2**
MH3D5691J8	-0,31	0,31	-0,11	0,19	0,01	0,12	0,01	0,17	-0,08	-0,06	0,15	-0,09	**31,31%**	**n=2**
UY3SJOMZ0L	0,08	0,04	0,30	-0,03	0,22	0,13	-0,03	0,05	-0,06	0,08	0,00	0,00	**30,20%**	**n=3**
AE6Y688F2R	0,24	0,28	0,38	-0,07	0,22	-0,05	-0,09	0,01	-0,08	-0,16	-0,25	-0,07	**38,33%**	**n=3**
7KJEF3LTX	0,13	0,17	-0,01	-0,07	0,14	0,30	0,18	0,07	-0,12	-0,16	0,05	0,05	**29,52%**	**n=6**
COF8R2203B	-0,17	-0,16	0,43	-0,13	-0,06	0,25	-0,25	0,20	-0,12	-0,14	0,27	-0,08	**42,71%**	**n=3**
6S9MY0L5F	0,20	0,02	0,01	0,23	0,12	0,12	-0,08	0,14	0,10	0,15	-0,13	-0,07	**23,00%**	**n=4**
7U1Y35A39H	0,08	-0,01	0,18	-0,05	-0,13	-0,01	0,01	-0,03	0,37	0,10	-0,12	-0,01	**36,75%**	**n=9**
H5COF8R220	0,37	0,39	0,39	0,30	0,21	0,19	0,19	0,25	0,17	0,04	0,16	0,11	**39,33%**	**n=3**
VZ4UD7HAS5	0,15	0,08	0,00	-0,02	0,11	0,16	0,18	0,13	-0,03	-0,11	-0,08	-0,08	**18,10%**	**n=7**
Z1KO2J052P	0,26	0,15	0,06	-0,10	-0,13	-0,06	0,19	0,08	0,09	0,00	-0,02	-0,05	**25,96%**	**n=1**
3Z08U7XBJN	-0,23	-0,06	-0,11	-0,09	0,11	0,08	0,04	-0,17	0,25	-0,27	0,01	0,20	**25,50%**	**n=9**
9S1957Y4UG	-0,11	-0,24	0,15	0,19	-0,47	-0,06	0,12	-0,11	-0,15	0,15	0,15	0,10	**19,34%**	**n=4**
8GW4ZM50YT	0,11	-0,05	-0,34	-0,30	0,13	0,15	0,15	-0,22	-0,28	-0,13	0,03	0,33	**32,94%**	**n=12**
DZL16N439	-0,23	-0,14	0,05	-0,29	0,14	0,10	-0,12	0,05	-0,03	-0,08	-0,04	0,01	**13,60%**	**n=5**
9H2C3NJRV	0,22	0,21	0,09	0,20	0,09	-0,05	-0,02	-0,01	-0,21	-0,11	-0,27	0,12	**21,97%**	**n=1**
60752XWRR6	0,07	0,36	0,04	0,10	-0,06	-0,23	-0,04	-0,22	0,07	-0,14	0,03	-0,10	**35,58%**	**n=2**
QCT3E875TP	0,13	0,02	-0,13	-0,11	0,26	0,20	0,38	0,02	-0,14	0,01	-0,08	0,11	**37,75%**	**n=7**
X845V37C86	0,18	-0,01	-0,02	0,12	0,35	-0,08	-0,03	-0,14	-0,01	0,01	-0,13	-0,03	**34,77%**	**n=5**
2ZH589F74U	-0,08	-0,22	0,21	-0,03	-0,09	0,27	0,07	-0,17	0,15	0,03	-0,22	-0,04	**26,68%**	**n=6**
16EJH8S331	0,31	0,08	0,04	0,23	0,20	0,00	-0,16	-0,13	-0,24	-0,09	-0,18	0,04	**30,71%**	**n=1**
4W6DD3GK	0,23	-0,05	-0,26	-0,26	0,16	0,31	0,07	0,07	-0,18	-0,07	0,11	0,02	**31,14%**	**n=6**
SN647UC3PB	0,09	-0,16	-0,06	-0,34	-0,10	0,17	0,02	-0,03	0,14	-0,16	0,00	0,19	**18,68%**	**n=12**
ZU192BQUDO	0,18	0,11	0,20	-0,14	0,19	0,08	-0,17	-0,21	-0,09	-0,10	-0,14	-0,12	**20,09%**	**n=3**
AE6Y688F2R	0,34	-0,10	-0,13	0,01	0,14	0,43	0,27	0,02	-0,10	-0,14	-0,06	0,08	**42,85%**	**n=6**
CYG47VE73L	0,45	0,29	0,08	0,03	-0,16	-0,02	-0,08	0,14	-0,03	-0,12	-0,17	0,01	**45,37%**	**n=1**
GK8EV3YLJ9	0,60	0,38	0,36	0,36	0,30	0,18	0,20	0,15	-0,01	-0,08	-0,04	0,00	**60,41%**	**n=1**
RM566SBFUZ	0,08	-0,13	-0,11	-0,33	-0,04	0,45	0,01	-0,14	-0,04	-0,29	0,00	0,27	**45,35%**	**n=6**
088W9145C3	-0,13	-0,16	-0,26	0,02	0,26	-0,03	0,21	0,00	-0,25	-0,18	0,00	0,31	**31,26%**	**n=12**
314F5479L7	0,42	0,19	0,20	0,09	0,05	-0,03	-0,07	-0,04	-0,12	-0,10	-0,14	-0,30	**42,20%**	**n=1**
5HGB102QU	0,90	0,82	0,69	0,60	0,54	0,46	0,41	0,32	0,26	0,19	0,13	0,07	**89,52%**	**n=1**
488HGB2L	0,50	0,39	0,07	-0,01	-0,09	-0,10	-0,09	0,01	-0,09	-0,06	-0,20	-0,07	**49,54%**	**n=1**
21H257G516	0,18	-0,12	-0,27	-0,28	0,35	0,35	0,34	0,05	-0,37	-0,21	0,04	0,24	**35,09%**	**n=6**
SW5QH8KX97	-0,17	0,37	-0,27	0,25	-0,10	0,17	-0,13	0,24	-0,18	0,14	0,00	0,20	**37,15%**	**n=2**
S69A7950PC	-0,43	0,00	-0,03	0,26	-0,26	0,05	0,02	0,17	-0,21	0,00	0,26	-0,27	**25,94%**	**n=4**
1F6715CG8	0,38	0,36	0,28	0,32	0,38	0,15	0,14	0,13	0,12	-0,07	0,05	-0,05	**38,23%**	**n=5**
75LOW81EU0	0,41	0,45	0,27	0,16	0,33	0,13	0,11	0,05	0,05	-0,06	0,04	-0,15	**44,69%**	**n=2**
C36S1524GT	0,10	-0,01	0,22	0,17	0,08	-0,09	-0,07	-0,04	-0,17	-0,06	-0,01	-0,09	**22,18%**	**n=3**
ZE16N119V0	0,27	0,02	-0,09	0,14	0,05	-0,09	0,17	0,03	-0,06	-0,36	-0,04	0,04	**27,11%**	**n=1**
0J16M008B9	-0,03	-0,17	-0,14	0,00	-0,03	0,07	0,12	0,06	-0,06	-0,29	0,02	0,17	**17,01%**	**n=12**
FQAMDXP442	-0,14	-0,20	-0,16	0,40	0,06	-0,25	-0,23	0,19	0,06	-0,01	-0,12	0,17	**39,59%**	**n=4**
0923X18E94	-0,36	-0,06	-0,09	0,20	-0,05	0,11	-0,33	0,19	-0,02	0,12	-0,18	0,11	**19,84%**	**n=4**
EAJN2H041V	0,09	-0,29	-0,31	0,10	-0,01	0,11	0,14	0,02	-0,17	-0,27	0,02	0,25	**25,10%**	**n=12**
4992D3NJ	0,37	0,27	0,36	0,27	0,25	0,12	0,03	0,05	-0,01	-0,21	-0,07	-0,09	**36,79%**	**n=1**
F47U263749	0,39	0,37	0,56	0,25	0,20	0,13	0,01	-0,04	-0,07	-0,13	-0,18	-0,21	**55,70%**	**n=3**
BW903DL7FK	0,07	0,00	0,00	-0,26	-0,01	0,04	-0,08	0,08	0,05	-0,04	0,17	-0,06	**17,14%**	**n=11**
57580O8JV	-0,04	-0,15	-0,02	-0,04	0,13	-0,01	-0,12	0,02	0,21	0,18	-0,18	-0,08	**20,99%**	**n=9**
RMLG314NVZ	-0,03	-0,05	-0,09	-0,05	0,24	-0,08	0,04	-0,06	-0,08	0,04	0,07	0,15	**24,42%**	**n=5**
93UUPW7ZW	-0,32	0,01	0,18	-0,22	-0,08	0,05	-0,08	-0,09	0,11	-0,11	0,00	0,06	**18,42%**	**n=3**
4UK7NA0MMH	-0,13	-0,20	-0,02	0,01	0,25	-0,03	-0,09	0,03	-0,08	0,03	-0,09	-0,06	**24,64%**	**n=5**
PD5G6RZ2MY	-0,34	0,14	-0,03	0,16	-0,13	0,27	-0,01	0,06	0,06	-0,13	0,04	0,16	**27,34%**	**n=6**
Y8K220ZVC3	-0,13	0,10	-0,21	-0,09	-0,14	-0,07	0,00	0,02	-0,01	0,16	-0,06	0,18	**17,78%**	**n=12**
FAA22G6T46	-0,25	0,01	-0,25	0,10	-0,22	0,34	-0,29	0,23	-0,24	0,08	-0,09	0,25	**34,17%**	**n=6**
7KJEE3LTX	0,11	0,06	0,57	-0,05	0,02	0,16	0,00	0,08	-0,02	0,15	0,16	-0,16	**56,83%**	**n=3**
6S1524GT5F	-0,07	-0,01	0,15	-0,10	-0,31	-0,04	-0,10	-0,14	-0,24	0,26	0,11	-0,03	**26,03%**	**n=10**
387FEZ314G	-0,09	-0,10	0,24	-0,04	0,06	-0,01	0,00	-0,14	-0,01	0,20	-0,09	0,00	**24,10%**	**n=3**
5F34580O8	-0,14	0,24	-0,02	0,03	0,17	-0,17	0,13	0,11	-0,19	0,18	-0,16	0,17	**24,46%**	**n=2**
KO2052P6B	0,24	-0,02	0,19	0,22	0,03	-0,03	-0,02	-0,09	-0,29	-0,10	-0,04	-0,05	**23,55%**	**n=1**
AMDQP4402A	0,12	-0,06	-0,13	-0,08	0,26	0,11	0,17	-0,02	-0,12	-0,08	-0,03	0,21	**25,52%**	**n=5**
6R7923X18E	0,10	0,14	0,04	0,20	-0,02	0,18	0,03	-0,03	-0,04	-0,13	0,01	0,27	**26,85%**	**n=12**

The autocorrelation factors are generally relatively low, which means that the demand is not strongly periodic. For example, the factors of the column n=12 indicate if there is seasonality in monthly demand. For almost all products, the factors is very low, between 0.00 and 0.15, confirming the relatively poor performance of seasonality based forecasting techniques.

Appendix 10: Effects of Forecasting Bias on the CSL

Case 1: Unbiased Forecast

Y (t)	Yp (t)	e (t)	SS (t)	e' (t)	OT ?	OS (t)
100	92	8	109	-9	1	9
100	105	-5	122	-22	1	22
100	109	-9	126	-26	1	26
100	100	0	117	-17	1	17
100	92	8	109	-9	1	9
100	87	13	104	-4	1	4

MAD	6,14	**OTD**	100%
SE	8,55	**AOS**	15

Case 2: Biased Forecast

Y (t)	Yp (t)	e (t)	SS (t)	e' (t)	OT ?	OS (t)
100	94	6	98	2	0	0
100	95	5	99	1	0	0
100	93	7	97	3	0	0
100	89	11	93	7	0	0
100	92	8	96	4	0	0
100	94	6	98	2	0	0

MAD	6,14	**OTD**	0%
SE	2,14	**AOS**	0

Notes:

Y (t)	Demand Quantity in Month t
Yp (t)	Demand Forecast Quantity for Month t
e (t)	Forecast Error (Y (t) - Yp (t))
SS (t)	Stock in Month t (here: Yp (t) + 2.01 * SE); with t (cf = 95%, df = 5, one-tailed) = 2.01
e' (t)	Effective Error (Y (t) - SS (t))
OT ?	On Time Delivery: Yes (1) or No (0)
OS (t)	Quantity Left on Stock on the End of Month t
MAD	Mean Absolute Deviation
SE	Standard Error of the Forecast (here computed from observed data, not as 1.25 * MAD)
OTD	On Time Delivery Rate in %
AOS	Average Quantity on Stock at Month End over the observed Period

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Definition of bias

Forecasting Method 1 is unbiased (/1\ The number of positive and negative forecasting deviations lies within the confidence interval [2 ; 4]; /2\ All forecasting deviations fall inside the confidence interval (95%) [-21.97 ; 21.97]).
Forecasting Method 2 is biased (/1\ The number of positive and negative forecasting deviations lies outside the confidence interval [2 ; 4] (Appendix 5); /2\ Almost all forecasting deviations fall outside the confidence interval (95%) [-5.49 ; 5.49]).

Results interpretation

Although both methods lead to the same Mean Absolute Deviation (MAD = 6.14), the measured Standard Error (2.14 < 8.55) is far lower for Method 2. This is not due here to a more accurate forecast, but to bias. All forecasting errors being positive, the variance and thus the standard deviation of the data is lower than it should be. This underestimates the inaccuracy of the forecast and leads to too low security stocks (computed as $t \cdot SE$, where t = 2.01 corresponding to a one-tailed alpha of 5% given df = 5), ultimately causing an OTD of 0.00%! In turn, the unbiased forecast example seems coherent with the expected OTD of 95%.

As discussed in Appendix 15, the relationship $SE = 1.25 \cdot MAD$ is true for unbiased forecasts with normal distributed errors. It would thus be applicable in Case 1 only, with a computed SE of 1.25 * 6.14 = 7.68. One will notice that the OTD then stays unchanged at 100%.

Appendix 11: Regression Analyses of Back-Order Factors

Panel A: Regression Analysis of DOH as a factor of Back-Orders

Regression Statistics	
Multiple R	0.66
R Square	**0.43**
Adjusted R Square	0.37
Standard Error	209.05
Observations	12

ANOVA

	df	*SS*	*MS*	*F*	*Sign. F*
Regression	1	328447	328447	8	0.0208
Residual	10	437028	43703		
Total	11	765475			

	Coefficients	*Std Err*	*t-Stat*	*P-Value*	*Lower 95%*	*Upper 95%*
Intercept	3810	1124	3.39	0.0069	1306	6315
DOH	**-85.76**	31.28	-2.74	0.0208	-155	-16

RESIDUALS

Observation	*Predicted Y*	*Residual*
1	791	-137
2	1031	-88
3	1083	270
4	805	-21
5	637	22
6	671	76
7	568	-122
8	680	-201
9	637	-162
10	757	-144
11	594	29
12	542	477

Panel B: Regression Analysis of Drop-In as a factor of Back-Orders

SUMMARY

Regression Statistics	
Multiple R	0.72
R Square	**0.52**
Adjusted R Square	0.47
Standard Error	192.19
Observations	12

ANOVA

	df	*SS*	*MS*	*F*	*Sign. F*
Regression	1	396103	396103	11	0.0084
Residual	10	369372	36937		
Total	11	765475			

	Coefficients	*Std Err*	*t-Stat*	*P-Value*	*Lower 95%*	*Upper 95%*
Intercept	-501	381	-1.315	0.218	-1349	348
Drop-In	**2.05**	0.63	3.275	0.008	1	3

RESIDUALS

Observation	*Predicted Y*	*Residual*
1	586	69
2	793	150
3	1019	334
4	705	79
5	608	50
6	736	11
7	403	43
8	465	15
9	814	-339
10	834	-222
11	855	-232
12	978	42

Panel C: Multiple Regression Analysis of DOH and Drop-In taken together

SUMMARY

Regression Statistics	
Multiple R	0.84
R Square	0.71
Adjusted R Square	**0.65**
Standard Error	156.16
Observations	12

ANOVA

	df	*SS*	*MS*	*F*	*Sign. F*
Regression	2	545990	272995	11	0.0036
Residual	9	219484	24387		
Total	11	765475			

	Coefficients	*Std Err*	*t-Stat*	*P-Value*	*Lower 95%*	*Upper 95%*
Intercept	1966	1042	1.887	0.092	-391	4324
DOH	**-61.37**	24.76	-2.48	0.035	-117	-5
Drop-In	**1.61**	0.54	2.99	0.015	0	3

RESIDUALS

Observation	*Predicted Y*	*Residual*
1	659	-4
2	994	-50
3	1208	145
4	762	22
5	566	92
6	691	56
7	356	90
8	484	-5
9	728	-253
10	830	-217
11	729	-106
12	789	231

The multiple regression equation can be stated as follows:

$$\hat{Y}_{BackOrder} = 1{,}966{,}000 - 61{,}370 \cdot DOH + 1.61 \cdot DropIn$$

with:		
	$\hat{Y}_{BackOrder}$	in Euro
	DOH	in days
	$DropIn$	in Euro

This means that for each additional day of DOH, back-orders go down by € 61,370, and for each additional Euro of drop-in, back-orders go up by € 1.61. The intercept tells us that with no inventory, back-orders would likely reach almost two million Euros.

However, this relationship can be altered by other factors responsible for the remaining 35% of back-order variation (e.g. production bottlenecks, etc.).

Appendix 12: Model Evaluation Product Lists

Panel A: Products selected for evaluation from July to December 2006

Filtering Criteria:	Ordered	Min	8
(Test Period:	MedianOT	Max	10
05-06 to 06-05)	OTD	Max	80%

98 Items

Item	Ordered	MedianOT	OTD
Y8K220ZVC3	12	4,0	58%
GK8EV3YLJ9	12	5,0	50%
WR7U1Y35A2	12	5,5	52%
9H05K228ZV	12	6,5	50%
C36S1524GT	12	7,5	62%
4DDY011F67	12	8,0	16%
H5COF8R220	12	8,0	31%
3B4367H526	12	8,0	42%
3ZRMLG314N	12	8,0	66%
AE6Y688F2R	12	8,0	75%
RM566SBFUZ	12	8,5	56%
QCT3E875TP	12	8,5	63%
X845V37C86	12	8,5	69%
3Z08U7XBJN	12	8,5	71%
8GW4ZM50YT	12	8,5	79%
8W2A45C39J	12	9,0	40%
16M338BWE3	12	9,0	58%
7T26244U5H	12	9,0	66%
088W9145C3	12	9,0	67%
0J16M008B9	12	9,0	68%
EAJN2H041V	12	9,0	69%
5HGB102QU	12	9,0	73%
16EJH8S331	12	9,0	76%
ADE3784627	12	9,0	77%
488HGB2L	12	9,0	78%
XB4V37CU6	12	9,0	78%
SN647UC3PB	12	9,0	80%
RDVHG8866R	12	9,0	80%
0923X18E94	12	9,5	47%
3B16W7Z2LO	12	9,5	54%
9HY2BO3119	12	9,5	62%
900146D4ZL	12	9,5	66%
16N119C0G1	12	9,5	67%
KOEJ052W6	12	9,5	70%
D203KSW3Q	12	9,5	77%
ZE16N119V0	12	9,5	79%
Z1KO2J052P	12	10,0	58%
6CBW203DSW	12	10,0	62%
FQAMDXP442	12	10,0	63%
2ZH589F74U	12	10,0	72%
4992D3NJ	11	5,0	29%
SW5QH8KX97	11	5,0	66%
PD5G6RZ2MY	11	5,0	71%
7KJEE3LTX	11	5,5	61%
57580O8JV	11	7,0	58%
AE6Y688F2R	11	7,0	69%
0EU3YR4DDY	11	7,0	77%
0U1F67A5CH	11	7,5	36%
F8R220ZBC3	11	8,0	47%
KO2052P6B	11	8,0	59%
ZU192BQUDO	11	8,0	66%
YKB9N331CX	11	8,5	7%
F47U263749	11	8,5	21%
9H2C3NJRV	11	8,5	55%
DZL16N439	11	8,5	71%
CYG47VE73L	11	8,5	72%
4W6DD3GK	11	9,0	67%
SW5QH8KX94	11	9,0	69%
314F5479L7	11	9,0	70%
3S42V6O4K3	11	9,0	72%
NQYC5X687D	11	9,0	73%
2QPK5N6RZD	11	9,5	51%
0CT3WJV9Q	11	9,5	70%
9S1957Y4UG	11	10,0	77%
X7D20X8B2	10	6,0	25%
5Q05B6ER7D	10	6,5	28%
BW903DL7FK	10	6,5	41%
AMDQP4402A	10	7,0	33%
H589G74N5Y	10	7,5	25%
7KJEF3LTX	10	8,0	22%
6S9MY0L5F	10	8,0	30%
21H257G516	10	8,0	80%
2Y7KKF314L	10	8,5	43%
UY3SJOMZ0L	10	8,5	45%
5F34580O8	10	8,5	46%
60752XWRR6	10	9,0	37%
1F6715CG8	10	9,0	68%
75LOW81EU0	10	9,0	71%
866QQ8C3WA	10	10,0	73%
7U1Y35A39H	9	7,0	23%
05K228ZVC3	9	7,0	66%
6S1524GT5F	9	7,0	80%
FAA22G6T46	9	7,5	41%
COF8R2203B	9	7,5	52%
4367H52631	9	8,0	21%
RMLG314NVZ	9	8,0	54%
4UK7NA0MMH	9	8,0	64%
3K5691Q8LX	9	8,5	35%
387FEZ314G	9	8,5	62%
VZ4UD7HAS5	9	9,0	35%
MH3D5691J8	9	9,0	63%
5X61AN64K	8	4,0	75%
S69A7950PC	8	4,5	50%
0TTH647UD3	8	5,0	41%
PBRXV4Z886	8	5,0	57%
6R7923X18E	8	6,5	41%
93UUPW7ZW	8	7,0	66%
E0CT3WQ4C	8	7,0	80%

Panel B: Products selected for evaluation from January to June 2007

Filtering Criteria:	Ordered	Min	8
(Test Period:	MedianOT	Max	10
05-12 to 06-11)	OTD	Max	80%

106 Items

Item	Ordered	MedianOT	OTD
GK8EV3YLJ9	12	6,0	70%
WR7U1Y35A2	12	7,5	50%
UY3SJOMZ0L	12	8,0	41%
3B4367H526	12	8,0	43%
H5COF8R220	12	8,0	48%
5HGB102QU	12	8,0	70%
4FUY3TC6GZ	12	8,0	70%
Q4422A4580	12	8,0	78%
9HY2BO3119	12	8,5	57%
3Z08U7XBJN	12	8,5	64%
X845V37C86	12	8,5	65%
4W6DD3GK	12	8,5	68%
488HGB2L	12	8,5	70%
7T26244U5H	12	9,0	50%
RM566SBFUZ	12	9,0	53%
9H2C3NJRV	12	9,0	62%
G74U500L53	12	9,0	67%
D3OKSW5RH8	12	9,0	70%
C36S1524GT	12	9,0	72%
LX97QE536S	12	9,0	75%
4DDY011F67	12	9,5	15%
02L96S72V7	12	9,5	25%
3S42V6O4K3	12	9,5	53%
P5KNNRZD8X	12	9,5	58%
2QPK5N6RZD	12	9,5	62%
50QC1POJ7M	12	9,5	64%
RDVHG8866R	12	9,5	66%
8Q0C89NZQ4	12	9,5	80%
088W9145C3	12	10,0	58%
A986PLS924	12	10,0	72%
R2809JVVQ	12	10,0	80%
9H05K228ZV	11	5,0	18%
6S9MY0L5F	11	6,5	30%
677VDHXBSE	11	6,5	58%
X7D20X8B2	11	7,0	42%
VHU8P9R09	11	7,5	29%
0U1F67A5CH	11	7,5	38%
QCT3E875TP	11	8,0	69%
46W37D863A	11	8,5	31%
3B16W7Z2LO	11	8,5	47%
0J16M008B9	11	8,5	56%
KO2052P6B	11	8,5	61%
EAJN2H041V	11	8,5	68%
3ZRMLG314N	11	8,5	69%
ZU192BQUDO	11	8,5	69%
16EJH8S331	11	8,5	72%
16M338BWE3	11	8,5	73%
19V7Y1KN9	11	8,5	74%
F8R220ZBC3	11	9,0	38%
KOEJ052W6	11	9,0	64%
NQYC5X687D	11	9,0	66%
314F5479L7	11	9,0	66%
SW5QH8KX94	11	9,0	70%
X4AN50ZU8W	11	9,0	75%
SN647UC3PB	11	9,0	79%
XB4V37CU6	11	9,0	79%
60752XWRR6	11	9,5	32%
FQAMDXP442	11	9,5	47%
2AJ6CH0K16	11	9,5	61%
L328AVD36S	11	9,5	62%
16N119C0G1	11	9,5	64%
9S1957Y4UG	11	9,5	65%
D203KSW3Q	11	9,5	68%
AE6Y688F2R	11	9,5	72%
Z1KO2J052P	11	9,5	76%
DZL16N439	11	9,5	77%
16243T5GFA	11	9,5	78%
ADE3784627	11	10,0	45%
PD5G6RZ2MY	10	4,0	58%
7KJEE3LTX	10	4,5	63%
57580O8JV	10	6,5	50%
011F6745C6	10	6,5	54%
5Q05B6ER7D	10	7,0	42%
H589G74N5Y	10	7,5	0%
YKB9N331CX	10	7,5	0%
FAA22G6T46	10	8,0	53%
8W2A45C39J	10	8,5	28%
75LOW81EU0	10	8,5	69%
G8R3203C43	10	8,5	77%
21H257G516	10	9,0	80%
67G5163Y7L	10	9,5	26%
JE203KTW3R	10	9,5	64%
5F34580O8	10	10,0	54%
HNLY94422H	10	10,0	71%
S69A7950PC	9	3,5	50%
4992D3NJ	9	4,0	35%
G479L73T49	9	4,5	54%
5X61AN64K	9	4,5	71%
6R7923X18E	9	5,0	53%
0TTH647UD3	9	5,5	30%
9442K3OKZD	9	6,0	20%
RMLG314NVZ	9	6,0	54%
AE6Y688F2R	9	6,0	63%
AMDQP4402A	9	6,5	33%
387FEZ314G	9	7,0	71%
2Y7KKF314L	9	8,0	41%
5W37JD96OJ	9	8,0	63%
4367H52631	9	8,5	0%
MH3D5691J8	9	9,0	45%
7U1Y35A39H	9	9,5	18%
4F6Q01L96Z	9	9,5	39%
61A664LT6	8	5,5	11%
0B7950PC1V	8	5,5	50%
BW903DL7FK	8	5,5	55%
PBRXV4Z886	8	6,0	42%
COF8R2203B	8	7,5	62%

Appendix 13: Best Method Allocation Sheet

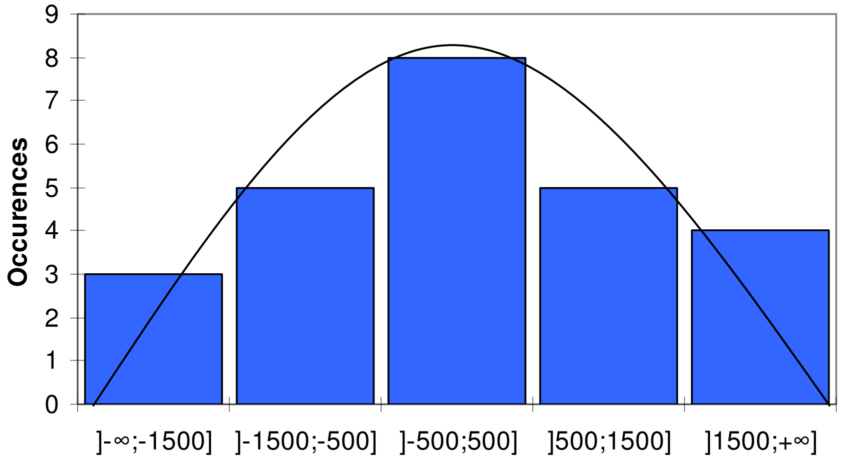
This is the result of the method reallocation made as of March 2007, prior to the computation of the demand forecasts for June 2007. The best method is the unbiased (i.e. bias index smaller than 3, see Appendix 5) method with the lowest MAD (MAD is computed over the prior 12 months, here April 06 to March 07).

Item	Exp. Smoothing		Moving Average		Naive Method		Best Method
	MAD	Bias	MAD	Bias	MAD	Bias	
5Q05B6ER7D	1.081	2	1.157	0	1.563	0	**ES**
AE6Y688F2R	292	3	417	0	292	3	**MA**
4DDY011F67	1.050	2	801	2	1.089	0	**MA**
677VDHXBSE	221	2	214	2	287	2	**MA**
67G5163Y7L	333	2	323	2	494	2	**MA**
VHU8P9R09	418	0	516	0	831	0	**ES**
8W2A45C39J	580	0	473	2	580	0	**MA**
COF8R2203B	69	2	70	0	67	3	**ES**
6S9MY0L5F	245	0	165	0	245	0	**MA**
7U1Y35A39H	273	2	307	2	389	0	**ES**
H5COF8R220	1.722	0	1.604	2	2.251	0	**MA**
0U1F67A5CH	708	0	687	0	799	0	**MA**
Z1KO2J052P	16	0	18	0	23	0	**ES**
3Z08U7XBJN	11	0	10	0	15	0	**MA**
L328AVD36S	6	3	7	3	11	2	**NM**
9S1957Y4UG	13	2	7	2	13	2	**MA**
D3OKSW5RH8	8	2	7	4	9	2	**ES**
8Q0C89NZQ4	12	3	11	0	12	3	**MA**
2AJ6CH0K16	15	0	16	0	20	0	**ES**
DZL16N439	10	2	11	3	15	2	**ES**
9H2C3NJRV	26	0	27	0	30	0	**ES**
60752XWRR6	11	2	11	0	13	0	**ES**
QCT3E875TP	20	1	19	0	26	0	**MA**
X845V37C86	12	0	11	0	15	0	**MA**
16EJH8S331	29	0	30	0	32	2	**ES**
4W6DD3GK	21	0	27	0	33	0	**ES**
SN647UC3PB	8	2	10	0	14	0	**ES**
Q4422A4580	14	0	13	0	15	2	**MA**
ZU192BQUDO	13	0	15	0	12	2	**NM**
AE6Y688F2R	33	2	31	2	44	2	**MA**
P5KNNRZD8X	85	0	86	0	95	0	**ES**
GK8EV3YLJ9	12	2	10	0	13	2	**MA**
RM566SBFUZ	9	0	9	0	14	0	**ES**
088W9145C3	10	0	9	0	12	0	**MA**
314F5479L7	12	0	14	0	11	2	**NM**
RDVHG8866R	34	0	37	0	43	0	**ES**
5HGB102QU	29	2	28	2	23	3	**MA**
488HGB2L	58	0	47	0	66	0	**MA**
21H257G516	28	0	26	0	31	0	**MA**
G479L73T49	30	0	27	2	39	3	**MA**
S69A7950PC	443	0	337	0	312	0	**NM**
19V7Y1KN9	19	0	20	0	25	2	**ES**
2Y7KKF314L	33	1	23	0	20	3	**MA**
75LOW81EU0	33	1	24	0	20	2	**NM**
C36S1524GT	26	0	26	0	28	2	**ES**
0J16M008B9	10	0	10	0	15	0	**MA**
FQAMDXP442	12	0	10	0	14	2	**MA**
EAJN2H041V	8	0	9	2	15	0	**ES**
4992D3NJ	601	0	467	1	342	3	**MA**
BW903DL7FK	168	2	215	0	108	3	**ES**
57580O8JV	168	0	170	0	162	3	**ES**
RMLG314NVZ	281	0	309	2	239	3	**ES**
4367H52631	805	0	726	0	1.026	0	**MA**
3B4367H526	699	2	789	0	800	0	**ES**
X7D20X8B2	226	2	224	0	342	0	**MA**
PBRXV4Z886	86	3	86	2	88	3	**MA**
4FUY3TC6GZ	120	1	108	1	116	2	**MA**
5W37JD96OJ	745	2	659	2	884	0	**MA**
011F6745C6	203	0	157	0	203	0	**MA**
4F6Q01L96Z	6	2	6	2	7	2	**MA**
PD5G6RZ2MY	2.304	0	2.126	0	2.442	0	**MA**
FAA22G6T46	57	0	51	2	80	0	**MA**
7KJEE3LTX	18	3	37	0	18	3	**MA**
387FEZ314G	29	0	48	0	75	2	**ES**
H589G74N5Y	576	2	622	0	594	2	**ES**
5F34580O8	27	0	23	0	24	0	**MA**
9H05K228ZV	1.142	2	1.220	2	1.191	2	**ES**
KO2052P6B	28	0	25	0	28	0	**MA**
AMDQP4402A	61	2	57	0	53	2	**NM**
6R7923X18E	41	0	41	0	50	0	**ES**
YKB9N331CX	86	2	68	2	116	0	**MA**
WR7U1Y35A2	20	0	19	0	24	2	**MA**

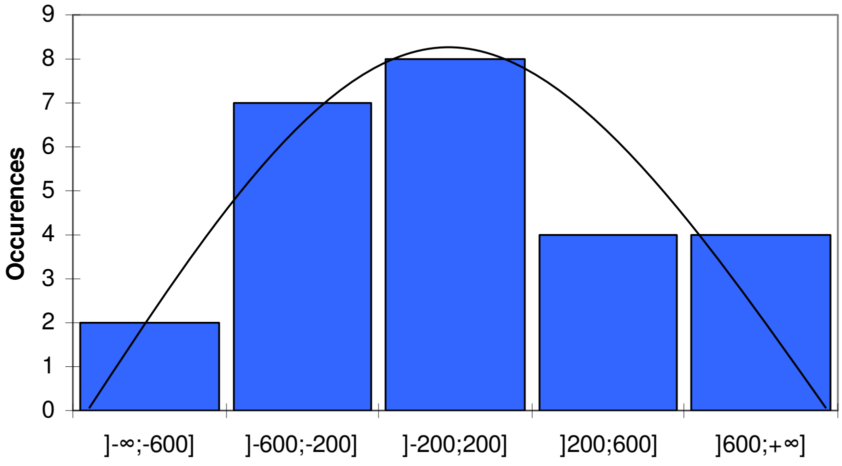
Note: ES=Exponential Smoothing; MA=Moving Average; NM=Naïve Method

Appendix 14: Examples of Histograms of Forecasting Errors

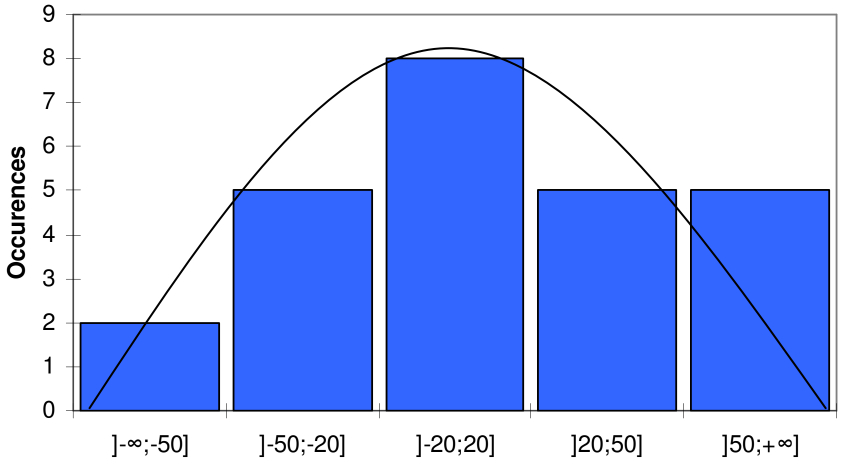
5Q05B6ER7D, Naive Method
(June 2005 to June 2007)



8W2A45C39J, Exponential Smoothing
(June 2005 to June 2007)

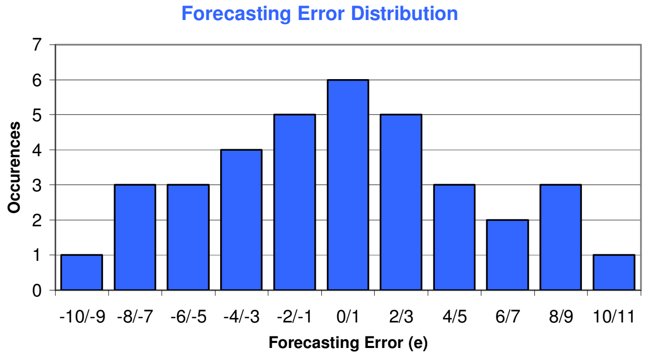


UY3SJOMZ0L, Four Months Moving Average
(June 2005 to June 2007)



Appendix 15: Relationship between the Mean Absolute Deviation and the Standard Error of the Forecast

t	Y (t)	Yp (t)	e (t)
1	50	40	10
2	37	45	-8
3	55	50	5
4	43	37	6
5	46	55	-9
6	40	43	-3
7	46	46	0
8	39	40	-1
9	47	46	1
10	42	39	3
11	43	47	-4
12	49	42	7
13	45	43	2
14	43	49	-6
15	44	45	-1
16	47	43	4
17	37	44	-7
18	47	47	0
19	45	37	8
20	45	47	-2
21	47	45	2
22	40	45	-5
23	51	47	4
24	38	40	-2
25	43	51	-8
26	38	38	0
27	46	43	3
28	47	38	9
29	43	46	-3
30	48	47	1
31	51	43	8
32	47	48	-1
33	53	51	2
34	42	47	-5
35	53	53	0
36	38	42	-4
		MAD:	4,00
		ME:	0,17
		SE:	5,03



Analysis:

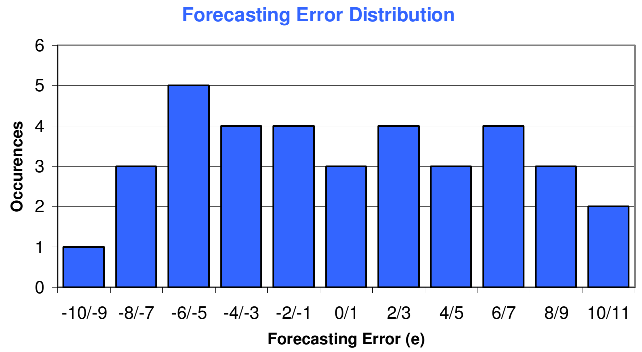
In this example, a two-months lagged naive method has been used to forecast demand: $Yp_{(t+2)} = Y_{(t)}$.

As the histogram shows, the forecasting errors appear to be normally distributed over the observed period of three years (36 months), with a mean error of about 0 (ME = 0.17). For a mean absolute deviation (MAD) of 4.00, a standard error (SE) of 5.03 has been computed.

The relationship between the MAD and SE is here: SE / MAD = 1.2575.

This instance illustrates the demonstrated relationship (Vollman, 1997) used in the optimization model: **SE = 1.25 * MAD**, whenever it can be assumed that the forecasting errors are normally distributed around 0.

t	Y (t)	Yp (t)	e (t)
1	50	47	3
2	46	51	-5
3	50	50	0
4	43	46	-3
5	49	50	-1
6	48	43	5
7	56	49	7
8	59	48	11
9	46	56	-10
10	67	59	8
11	52	46	6
12	73	67	6
13	44	52	-8
14	72	73	-1
15	41	44	-3
16	65	72	-7
17	41	41	0
18	67	65	2
19	48	41	7
20	63	67	-4
21	57	48	9
22	57	63	-6
23	62	57	5
24	50	57	-7
25	60	62	-2
26	51	50	1
27	68	60	8
28	61	51	10
29	64	68	-4
30	64	61	3
31	59	64	-5
32	62	64	-2
33	61	59	2
34	57	62	-5
35	55	61	-6
36	61	57	4
		MAD:	4,89
		ME:	0,50
		SE:	5,74



Analysis:

Again, a two-months lagged naive method has been used to forecast demand: $Yp_{(t+2)} = Y_{(t)}$.

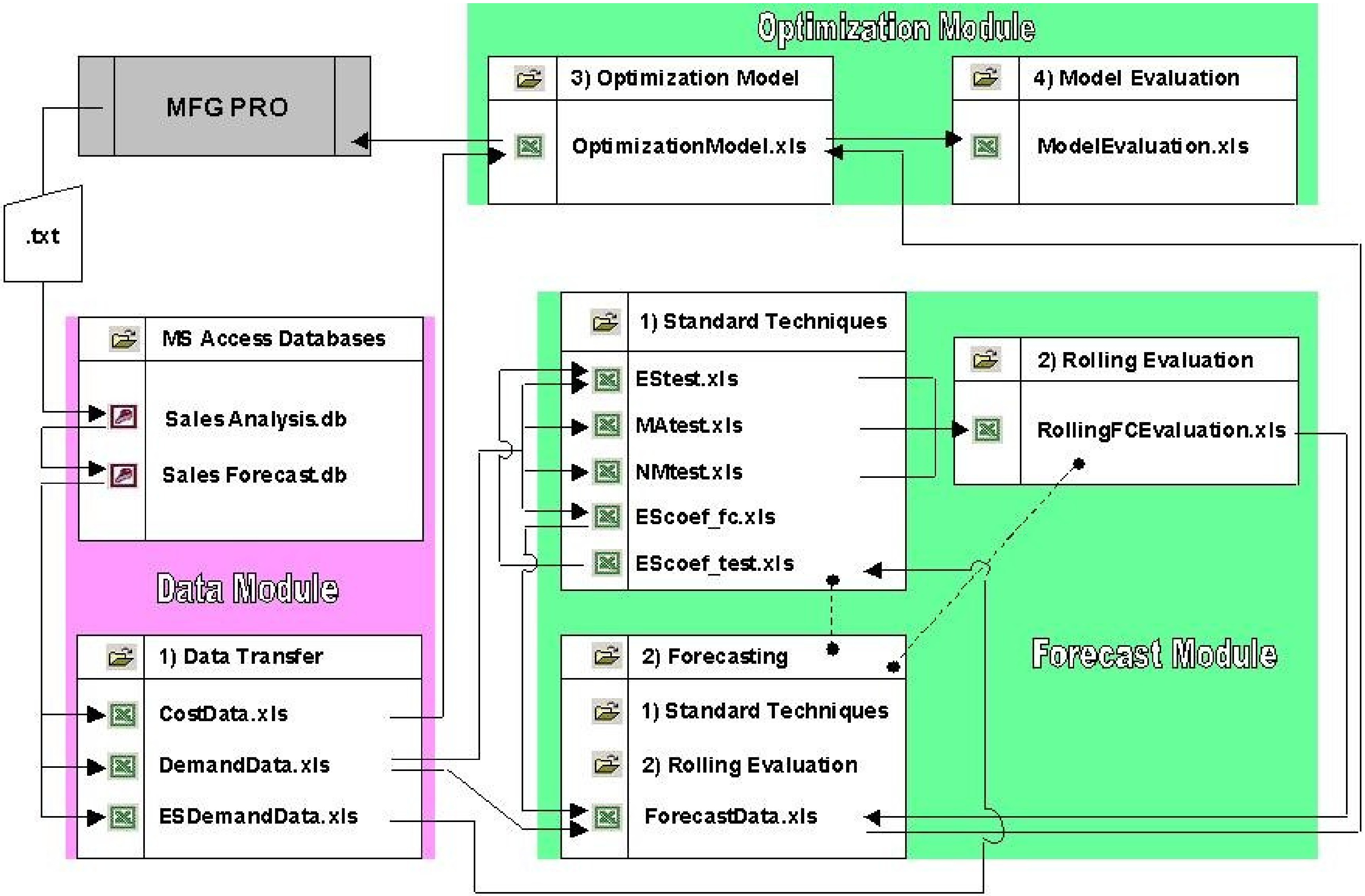
However, the forecasting errors over this period of three years (36 months), though having a mean error close to 0 (ME = 0.50), do not as clearly follow the shape of a Gauss curve. For a mean absolute deviation (MAD) of 4.89, a standard error (SE) of 5.74 has been computed.

The relationship between the MAD and SE is here: SE / MAD = 1.1738.

For this sample, the priorly enounced relationship is not exactly verified. In fact, the actual SE is smaller than the one that would be expected from a normally distribution sample of forecasting errors. However, whenever the sample distribution does not clearly speak against normal distribution around a mean of 0, it can still be assumed that the population of forecasting errors is normally distributed, hence SE = 1.25 * MAD.

It must be stressed that the above mentioned relationship is thus wrong whenever a forecasting method produces biased forecasts (ME significantly different from 0).

Appendix 16: Detailed System Structure



Appendix 17: Userform Sample Masks

These are only two examples of user forms as they appear throughout the optimization system. The yellow button on the top-right corner of the Excel forms is a link to the help file. The help file explain in detail what is to be done in the concerned file.

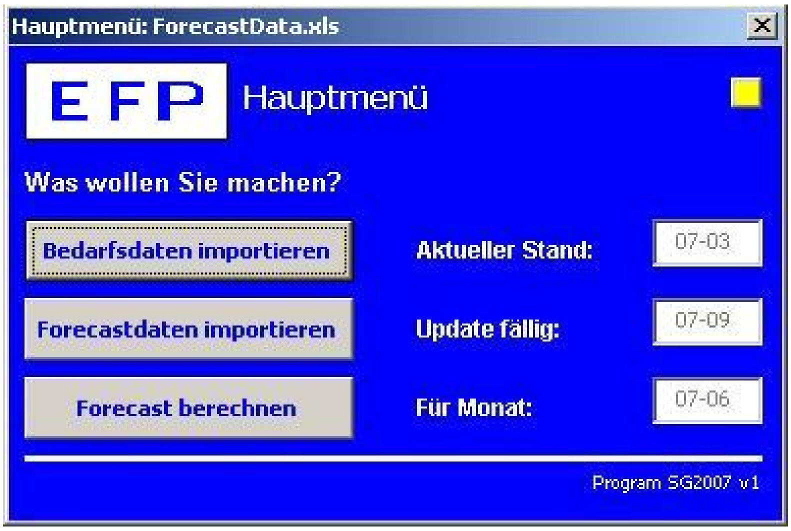
Main Menu of RollingFCEvaluation Excel File:

1. Import Forecast Data
2. Determine Best Method for each product
3. Allocate Forecast Data accordingly

Typ: ● Automatisch ○ Manuell

Main Menu of ForecastData Excel File:

1. Import Demand Data (from DemandData)
2. Import Forecast Data (from RollingFCEvaluation)
3. Compute Forecasts



Aktueller Stand: 07-03
Update fällig: 07-09
Für Monat: 07-06

Appendix 18: Extract of the User's Handbook

The following is a description of the steps the user needs to do in the Data Module after having downloaded the demand and cost data from the MS Access Database 'Sales Forecast' to the respective target files. The handbook is written in German language for the users' convenience.

Data Module (DM)

[Zuvor wurden alle Bedarfs- und Kostdaten aus der Access-Datenbank geladen]

DemandData.xls

Folgende Schritte dauern zk. 30 Sekunden:

- Öffnen Sie die Datei
- Drücken Sie im Hauptmenü auf |Neuen Bericht erstellen|, falls es das zu bearbeitende Berichtblatt noch nicht gibt
- Geben Sie den gewünschten Berichtnamen und den Berichtszeitraum an
- Drücken Sie auf |Bericht erstellen| → ein neues Berichtblatt wird von den aus Access geladenen Daten erstellt
- Drücken Sie im Hauptmenü auf |Bedarfsdaten bereiten|
- Geben Sie den Bericht an, welchen Sie formatieren wollen
- Drücken Sie auf |Daten laden| → die Daten des Berichts werden auf das Transferblatt ‚UL_DemandData' geladen und formatiert
- Schließen Sie die Datei

ESDemandData.xls

Folgende Schritte dauern zk. 30 Sekunden:

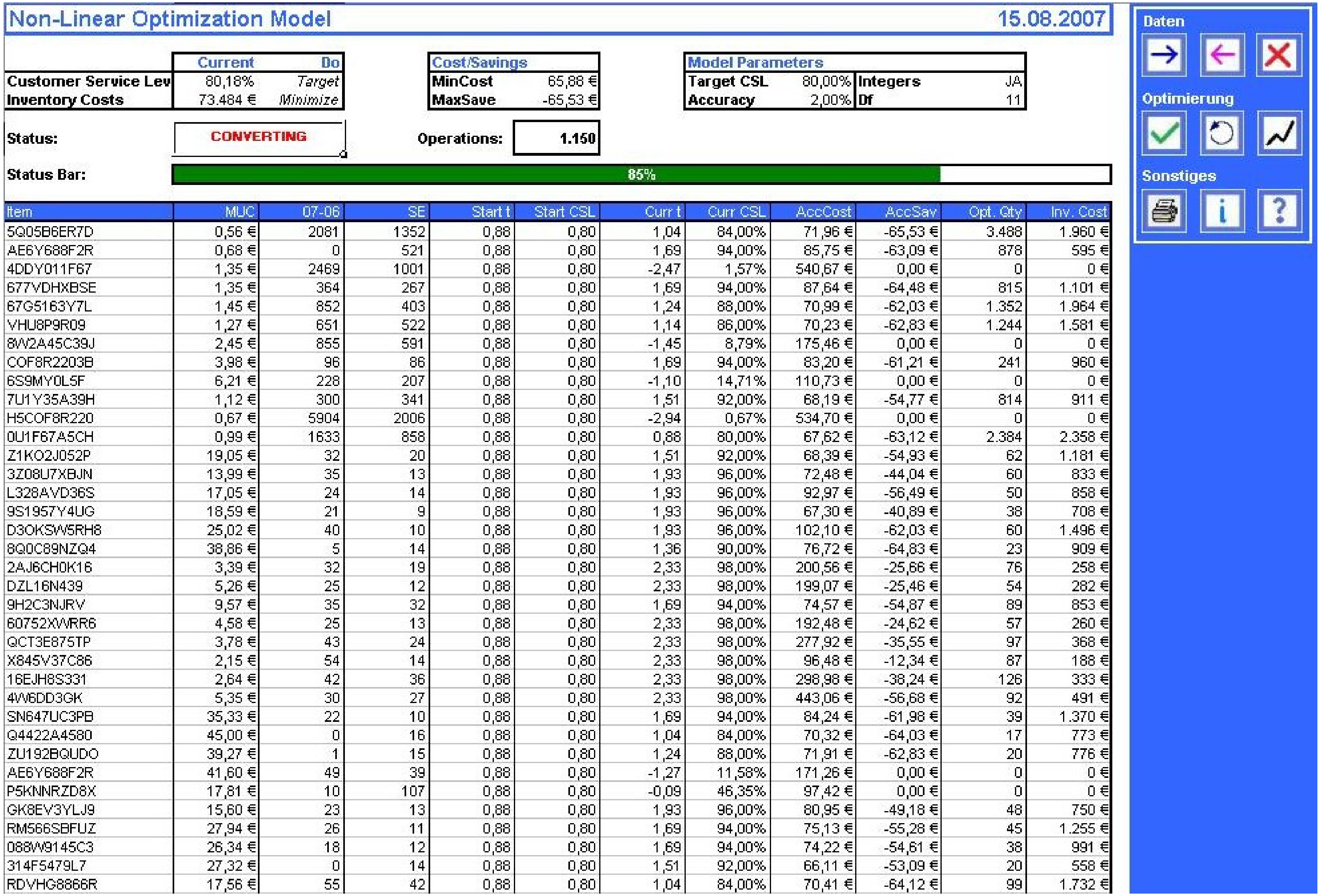
- Öffnen Sie die Datei
- Drücken Sie im Hauptmenü auf |Neuen Bericht erstellen|, falls es das zu bearbeitende Berichtblatt noch nicht gibt
- Geben Sie den gewünschten Berichtnamen und den Berichtszeitraum an
- Drücken Sie auf |Bericht erstellen| → ein neues Berichtblatt wird von den aus Access geladenen Daten erstellt
- Drücken Sie im Hauptmenü auf |Bedarfsdaten bereiten|
- Geben Sie den Bericht an, welchen Sie formatieren wollen
- Drücken Sie auf |Daten laden| → die Daten des Berichts werden auf das Transferblatt ‚UL_ESDemandData' geladen und formatiert
- Schließen Sie die Datei

CostData.xls

- Öffnen Sie die Datei
- Prüfen Sie, ob die Kostdaten der aktuell für das Modell selektierten Produkte korrekt auf das Blatt ‚DL_CostData' geladen wurden (wenn nicht, laden Sie diese Daten bis aus der MS Access Datenbank ‚Sales Forecast'
- Schließen Sie die Datei

Appendix 19: Optimization Model User Interface

The following is a screen shot of the file ‘OptimizationModel’ taken during the optimization process. The command buttons on the right-hand side have the following functions, respectively from top to bottom and left to right: download forecasts and cost data, upload optimized material quantities, erase current data, optimize, initialize, transfer data to cost/service graph, print, show integer optimization sheet, and help.



Non-Linear Optimization Model 15.08.2007

	Current	Do
Customer Service Lev	80,18%	*Target*
Inventory Costs	73.484 €	*Minimize*

Cost/Savings	
MinCost	65,88 €
MaxSave	-65,53 €

Model Parameters			
Target CSL	80,00%	Integers	JA
Accuracy	2,00%	Df	11

Status: CONVERTING

Operations: 1.150

Status Bar: 85%

Item	MUC	07-06	SE	Start t	Start CSL	Curr t	Curr CSL	AccCost	AccSav	Opt. Qty	Inv. Cost
5Q05B6ER7D	0,56 €	2081	1352	0,88	0,80	1,04	84,00%	71,96 €	-65,53 €	3.488	1.960 €
AE6Y688F2R	0,68 €	0	521	0,88	0,80	1,69	94,00%	85,75 €	-63,09 €	878	595 €
4DDY011F67	1,35 €	2469	1001	0,88	0,80	-2,47	1,57%	540,67 €	0,00 €	0	0 €
677VDHXBSE	1,35 €	364	267	0,88	0,80	1,69	94,00%	87,64 €	-64,48 €	815	1.101 €
67G5163Y7L	1,45 €	852	403	0,88	0,80	1,24	88,00%	70,99 €	-62,03 €	1.352	1.964 €
VHU8P9R09	1,27 €	651	522	0,88	0,80	1,14	86,00%	70,23 €	-62,83 €	1.244	1.581 €
8W2A45C39J	2,45 €	855	591	0,88	0,80	-1,45	8,79%	175,46 €	0,00 €	0	0 €
COF8R2203B	3,98 €	96	86	0,88	0,80	1,69	94,00%	83,20 €	-61,21 €	241	960 €
6S9MY0L5F	6,21 €	228	207	0,88	0,80	-1,10	14,71%	110,73 €	0,00 €	0	0 €
7U1Y35A39H	1,12 €	300	341	0,88	0,80	1,51	92,00%	68,19 €	-54,77 €	814	911 €
H5COF8R220	0,67 €	5904	2006	0,88	0,80	-2,94	0,67%	534,70 €	0,00 €	0	0 €
0U1F67A5CH	0,99 €	1633	858	0,88	0,80	0,88	80,00%	67,62 €	-63,12 €	2.384	2.358 €
Z1KO2J052P	19,05 €	32	20	0,88	0,80	1,51	92,00%	68,39 €	-54,93 €	62	1.181 €
3Z08U7XBJN	13,99 €	35	13	0,88	0,80	1,93	96,00%	72,48 €	-44,04 €	60	833 €
L328AVD36S	17,05 €	24	14	0,88	0,80	1,93	96,00%	92,97 €	-56,49 €	50	858 €
9S1957Y4UG	18,59 €	21	9	0,88	0,80	1,93	96,00%	67,30 €	-40,89 €	38	708 €
D3OKSW5RH8	25,02 €	40	10	0,88	0,80	1,93	96,00%	102,10 €	-62,03 €	60	1.496 €
8Q0C89NZQ4	38,86 €	5	14	0,88	0,80	1,36	90,00%	76,72 €	-64,83 €	23	909 €
2AJ6CH0K16	3,39 €	32	19	0,88	0,80	2,33	98,00%	200,56 €	-25,66 €	76	258 €
DZL16N439	5,26 €	25	12	0,88	0,80	2,33	98,00%	199,07 €	-25,46 €	54	282 €
9H2C3NJRV	9,57 €	35	32	0,88	0,80	1,69	94,00%	74,57 €	-54,87 €	89	853 €
60752XWRR6	4,58 €	25	13	0,88	0,80	2,33	98,00%	192,48 €	-24,62 €	57	260 €
QCT3E875TP	3,78 €	43	24	0,88	0,80	2,33	98,00%	277,92 €	-35,55 €	97	368 €
X845V37C86	2,15 €	54	14	0,88	0,80	2,33	98,00%	96,48 €	-12,34 €	87	188 €
16EJH8S331	2,64 €	42	36	0,88	0,80	2,33	98,00%	298,98 €	-38,24 €	126	333 €
4W6DD3GK	5,35 €	30	27	0,88	0,80	2,33	98,00%	443,06 €	-56,68 €	92	491 €
SN647UC3PB	35,33 €	22	10	0,88	0,80	1,69	94,00%	84,24 €	-61,98 €	39	1.370 €
Q4422A4580	45,00 €	0	16	0,88	0,80	1,04	84,00%	70,32 €	-64,03 €	17	773 €
ZU192BQUDO	39,27 €	1	15	0,88	0,80	1,24	88,00%	71,91 €	-62,83 €	20	776 €
AE6Y688F2R	41,60 €	49	39	0,88	0,80	-1,27	11,58%	171,26 €	0,00 €	0	0 €
P5KNNRZD8X	17,81 €	10	107	0,88	0,80	-0,09	46,35%	97,42 €	0,00 €	0	0 €
GK8EV3YLJ9	15,60 €	23	13	0,88	0,80	1,93	96,00%	80,95 €	-49,18 €	48	750 €
RM566SBFUZ	27,94 €	26	11	0,88	0,80	1,69	94,00%	75,13 €	-55,28 €	45	1.255 €
088W9145C3	26,34 €	18	12	0,88	0,80	1,69	94,00%	74,22 €	-54,61 €	38	991 €
314F5479L7	27,32 €	0	14	0,88	0,80	1,51	92,00%	66,11 €	-53,09 €	20	558 €
RDVHG8866R	17,56 €	55	42	0,88	0,80	1,04	84,00%	70,41 €	-64,12 €	99	1.732 €

Appendix 20: Model Evaluation Results

Panel A: Methodology of the Model Evaluation

In order to conduct the hypothesis test presented in section 6.1.1 of this paper, the monthly customer service levels that would have been obtained using the model over the period from July 2006 to June 2007 have been simulated.

For each month of this period, the simulation was done based on the data that were available three months earlier, as shown in the following table. The optimized quantities were then compared to the actual demand for the month, and the CSL computed according to each of the two definitions (see 2.1.4). For the simulation, the best forecasting method allocation was performed monthly. The product scope was revised after the first six months, as shown by the product lists in Appendix 12. The simulation results are displayed in Panel B.

Forecast for:	As of:	Demand over:	Alpha Optim.:	Forecast Test:
06-07	06-04	04-05 / 06-04	03-05 / 05-04	05-05 / 06-04
06-08	06-05	04-06 / 06-05	03-06 / 05-05	05-06 / 06-05
06-09	06-06	04-07 / 06-06	03-07 / 05-06	05-07 / 06-06
06-10	06-07	04-08 / 06-07	03-08 / 05-07	05-08 / 06-07
06-11	06-08	04-09 / 06-08	03-09 / 05-08	05-09 / 06-08
06-12	06-09	04-10 / 06-09	03-10 / 05-09	05-10 / 06-09
07-01	06-10	04-11 / 06-10	03-11 / 05-10	05-11 / 06-10
07-02	06-11	04-12 / 06-11	03-12 / 05-11	05-12 / 06-11
07-03	06-12	05-01 / 06-12	04-01 / 05-12	06-01 / 06-12
07-04	07-01	05-02 / 07-01	04-02 / 06-01	06-02 / 07-01
07-05	07-02	05-03 / 07-02	04-03 / 06-02	06-03 / 07-02
07-06	07-03	05-04 / 07-03	04-04 / 06-03	06-04 / 07-03

Forecast for	Month for which the forecast is simulated
As of	Last known demand month included in the forecast
Demand over	Demand data used for the simulation
Alpha Optim.	Twenty-four month period over which the smoothing factor Alpha is optimized for the assessment of the accuracy and honesty of the exponential smoothing forecasting method
Forecast Test	Twelve month period over which the accuracy and honesty of the three forecasting techniques (exponential smoothing, moving average, and naïve method) are compared for the best method allocation

Panel B: CSL Simulation Results

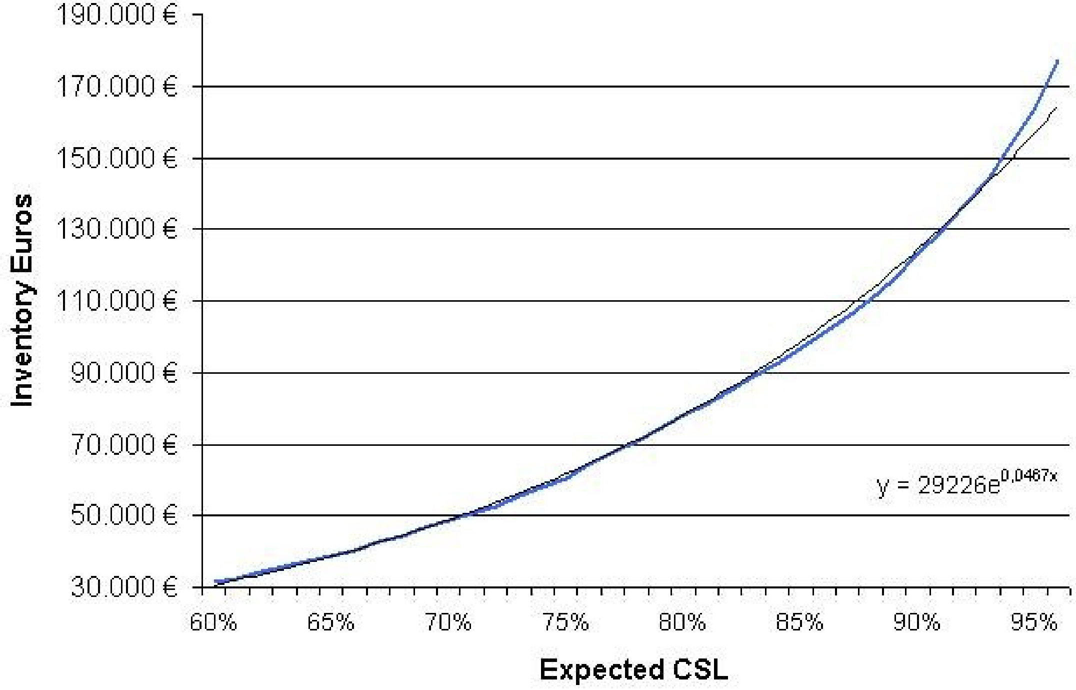
	OTD by order (as reported, see equation 2)		OTD of total monthly demand (as computed by the model)		
Month	**Real CSL**	**Sim CSL**	**Real CSL**	**Sim CSL**	**Target CSL**
06-07	61%	89%	33%	86%	80%
06-08	66%	84%	55%	94%	80%
06-09	55%	86%	31%	83%	80%
06-10	67%	81%	34%	72%	80%
06-11	64%	74%	32%	61%	80%
06-12	76%	79%	49%	73%	80%
07-01	74%	77%	66%	81%	80%
07-02	76%	85%	53%	87%	80%
07-03	68%	85%	45%	84%	80%
07-04	49%	86%	23%	88%	80%
07-05	61%	84%	43%	84%	80%
07-06	49%	84%	24%	84%	80%
Mean	63,83%	82,83%	40,67%	81,41%	80%

Appendix 21: Cost / Service Analysis

Panel A: Average Monthly Material Investment per target CSL

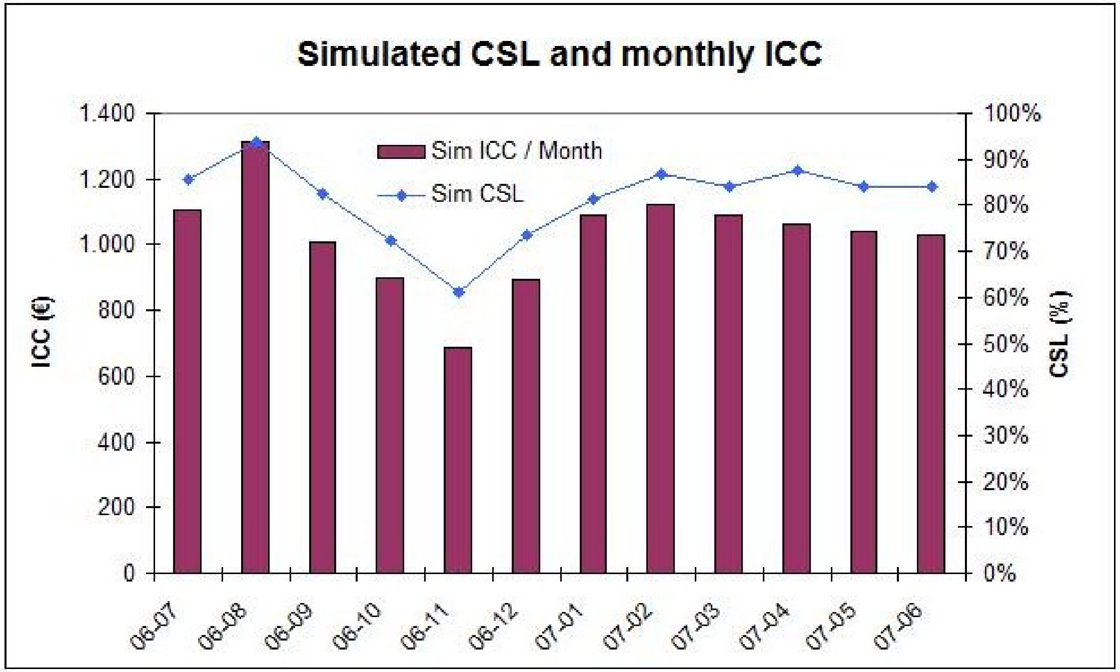
CSL	Min.Cost
60%	31.277,71 €
61%	32.327,06 €
62%	34.397,58 €
63%	36.036,75 €
64%	37.486,25 €
65%	38.948,84 €
66%	40.282,17 €
67%	42.807,47 €
68%	44.333,90 €
69%	46.890,32 €
70%	48.878,14 €
71%	50.626,94 €
72%	52.516,89 €
73%	55.640,98 €
74%	58.011,89 €
75%	60.284,04 €
76%	64.514,90 €
77%	67.892,92 €
78%	70.775,39 €
79%	74.302,87 €
80%	78.080,21 €
81%	80.967,88 €
82%	84.804,40 €
83%	88.875,71 €
84%	92.304,23 €
85%	96.990,43 €
86%	101.181,99 €
87%	105.524,82 €
88%	110.277,04 €
89%	115.917,48 €
90%	123.103,14 €
91%	129.192,25 €
92%	136.996,63 €
93%	144.038,24 €
94%	153.513,17 €
95%	163.820,89 €
96%	177.187,15 €

Accuracy:	2,00%		
Df:	11		
Integers:	JA	Period:	July 2006 to June 2007



Panel B: **Simulated ICC for a target CSL of 80%**

Month	# Products	Target CSL	Sim CSL	Start Inv. I_1	End Inv. I_2	Average Inv.	Capital Return r	Sim ICC / Month
06-07	98	80,00%	85,71%	87.648 €	44.757 €	66.203 €	1,67%	1.103 €
06-08	98	80,00%	93,88%	84.712 €	72.704 €	78.708 €	1,67%	1.312 €
06-09	98	80,00%	82,65%	85.417 €	35.303 €	60.360 €	1,67%	1.006 €
06-10	98	80,00%	72,45%	78.365 €	29.316 €	53.841 €	1,67%	897 €
06-11	98	80,00%	61,22%	63.280 €	19.253 €	41.267 €	1,67%	688 €
06-12	98	80,00%	73,47%	72.902 €	34.068 €	53.485 €	1,67%	891 €
07-01	106	80,00%	81,13%	75.821 €	55.225 €	65.523 €	1,67%	1.092 €
07-02	106	80,00%	86,79%	82.407 €	52.237 €	67.322 €	1,67%	1.122 €
07-03	106	80,00%	83,96%	82.867 €	47.717 €	65.292 €	1,67%	1.088 €
07-04	106	80,00%	87,74%	75.041 €	52.694 €	63.868 €	1,67%	1.064 €
07-05	106	80,00%	83,96%	75.033 €	50.144 €	62.589 €	1,67%	1.043 €
07-06	106	80,00%	83,96%	73.470 €	50.293 €	61.882 €	1,67%	1.031 €
Mean/Sum	102	80,00%	81,41%	78.080 €	45.309 €	61.695 €	20%	12.339 €



From this table, we derive that on average, 42% of the start inventory have been sold during the month, i.e. $I_2 = 0.68\ I_1$.

Panel C: **Regression Analysis of the CSL/ICC Relationship for a target CSL of 80%**

SUMMARY

Regression Statistics	
Multiple R	0.96
R Square	0.92
Adjusted R Square	0.91
Standard Error	44.70
Observations	12

COMMENTARY

- High correlation between ICC and CSL (corr. Factor of 0,96)
- 92,21% of the variation in ICC is explained by the variation in CSL
- Random residual plot (no autocorrelation)

\>> Regression line: **ICC** = 1,703 * **CSL** - 358

i.e. each additional % of CSL increases ICC by € 17 (with ICC=0, CSL remains at 21%)

(assumption is an annual capital return rate of R = 20%)

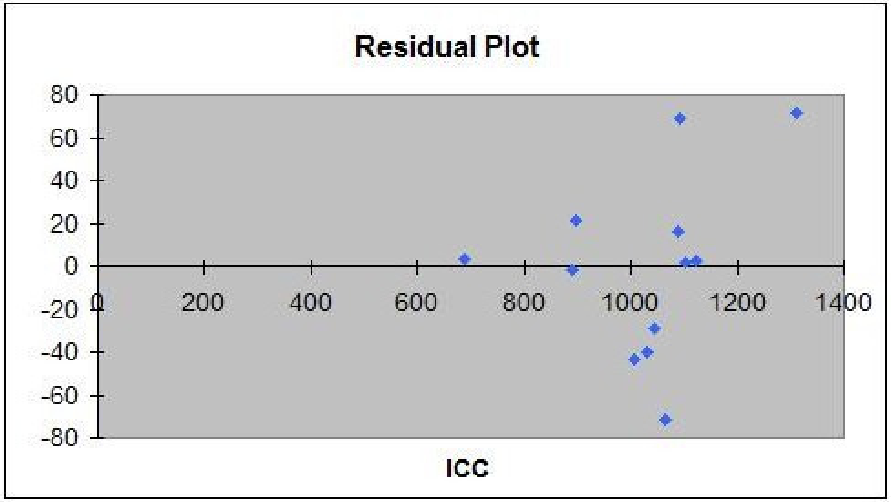
ANOVA

	df	*SS*	*MS*	*F*	*Sign. F*
Regression	1	236,345.08	236,345.08	118.29	7.32E-07
Residual	10	19,980.15	1,998.02		
Total	11	256,325.23			

	Coefficients	*Std Err*	*t-Stat*	*P-Value*	*Lower 95%*	*Upper 95%*
Intercept	-358.03	128.11	-2.79	0.019	-643.48	-72.58
CSL	1,702.83	156.57	10.88	7.324E-07	1,353.98	2,051.68

RESIDUALS

Observation	*Predicted Y*	*Residual*
1	1101	2
2	1241	71
3	1049	-43
4	876	22
5	684	3
6	893	-2
7	1023	69
8	1120	2
9	1072	17
10	1136	-72
11	1072	-29
12	1072	-40



Appendix 22: Monte Carlo Simulation of Inventory Costs given a Random Material Quantity Allocation respecting the Model Constraints (Example of July 2006)

Panel A: Example Output of Iteration N°50

Target CSL:	**60,00%**
Expected CSL:	**60,01%**

Tot. Inv. Cost	**124.096 €**

Iterationen	**50**

Item	MUC	FC 06-07	SE	Rand. Qty	t-Value	CSL	Inv. Cost
5Q05B6ER7D	0,68 €	3700	917	4012	0,34	63%	2.719,77 €
4DDY011F67	1,35 €	3132	975	3411	0,29	61%	4.607,68 €
3K5691Q8LX	2,59 €	261	149	269	0,05	52%	696,83 €
8W2A45C39J	3,12 €	750	341	955	0,60	72%	2.977,06 €
0U1F67A5CH	1,12 €	1215	808	1534	0,40	65%	1.716,37 €
2Y7KKF314L	15,55 €	62	25	61	-0,05	48%	940,92 €
4367H52631	2,79 €	976	491	1143	0,34	63%	3.190,33 €
3B4367H526	1,05 €	1983	742	2059	0,10	54%	2.158,56 €
X7D20X8B2	1,70 €	592	331	592	0,00	50%	1.004,23 €
PBRXV4Z886	3,12 €	206	71	206	0,00	50%	644,47 €
H589G74N5Y	3,85 €	1400	485	1772	0,77	77%	6.823,70 €
9H05K228ZV	3,06 €	1983	477	2094	0,23	59%	6.404,25 €
YKB9N331CX	8,63 €	133	72	150	0,23	59%	1.294,37 €
0TTH647UD3	0,24 €	499	118	527	0,23	59%	128,31 €
MH3D5691J8	2,22 €	907	874	1278	0,42	66%	2.836,40 €
UY3SJOMZ0L	6,48 €	15	27	25	0,37	64%	161,32 €
AE6Y688F2R	0,70 €	500	312	623	0,40	65%	435,12 €
7KJEF3LTX	0,96 €	657	312	763	0,34	63%	734,63 €
COF8R2203B	4,25 €	6	86	85	0,91	81%	360,33 €
6S9MY0L5F	7,10 €	200	191	205	0,03	51%	1.453,90 €
7U1Y35A39H	1,14 €	623	306	719	0,31	62%	816,75 €
H5COF8R220	0,80 €	3600	1823	4171	0,31	62%	3.321,75 €
VZ4UD7HAS5	6,30 €	0	1	0	0,21	58%	2,84 €
Z1KO2J052P	13,83 €	39	13	41	0,15	56%	563,03 €
3Z08U7XBJN	13,99 €	28	13	32	0,29	61%	444,19 €
9S1957Y4UG	18,59 €	25	11	27	0,21	58%	506,86 €
8GW4ZM50YT	43,28 €	28	19	41	0,66	74%	1.766,27 €
DZL16N439	4,87 €	24	13	25	0,08	53%	121,03 €
9H2C3NJRV	9,57 €	59	21	77	0,84	79%	732,97 €
60752XWRR6	5,01 €	26	17	29	0,18	57%	145,34 €
QCT3E875TP	4,95 €	50	15	50	0,03	51%	247,43 €
X845V37C86	2,10 €	40	21	45	0,21	58%	93,68 €
2ZH589F74U	12,28 €	14	4	15	0,23	59%	184,47 €
16EJH8S331	2,64 €	51	15	58	0,45	67%	152,35 €
4W6DD3GK	5,35 €	30	11	29	-0,05	48%	157,00 €
SN647UC3PB	33,80 €	27	9	25	-0,26	40%	848,71 €
ZU192BQUDO	39,27 €	20	7	23	0,45	67%	899,54 €
AE6Y688F2R	39,58 €	500	312	556	0,18	57%	22.019,85 €
CYG47VE73L	42,05 €	68	26	78	0,40	65%	3.272,32 €
GK8EV3YLJ9	15,60 €	26	9	30	0,57	71%	475,15 €
RM566SBFUZ	25,83 €	21	8	23	0,23	59%	599,80 €
088W9145C3	26,67 €	25	10	29	0,40	65%	768,50 €
314F5479L7	27,32 €	26	11	31	0,48	68%	854,48 €
5HGB102QU	19,73 €	38	20	44	0,31	62%	874,96 €
488HGB2L	22,53 €	133	37	139	0,18	57%	3.140,99 €
21H257G516	39,95 €	28	17	35	0,40	65%	1.384,91 €
SW5QH8KX97	5,70 €	316	129	357	0,31	62%	2.032,71 €
S69A7950PC	1,81 €	508	515	669	0,31	62%	1.212,79 €
1F6715CG8	32,15 €	27	18	34	0,37	64%	1.079,34 €
75LOW81EU0	19,16 €	61	24	66	0,23	59%	1.273,16 €
C36S1524GT	21,52 €	57	21	54	-0,13	45%	1.158,84 €
ZE16N119V0	23,84 €	43	23	44	0,05	52%	1.048,11 €
0J16M008B9	15,58 €	28	7	31	0,42	66%	476,44 €
FQAMDXP442	22,10 €	18	8	19	0,15	56%	424,51 €
0923X18E94	19,93 €	23	14	29	0,42	66%	574,18 €
EAJN2H041V	19,64 €	27	8	30	0,29	61%	581,94 €
4992D3NJ	0,62 €	1077	313	1245	0,54	70%	775,51 €
F47U263749	1,11 €	1922	689	2552	0,91	81%	2.824,56 €
BW903DL7FK	1,58 €	378	130	365	-0,10	46%	578,09 €
57580O8JV	2,22 €	383	192	443	0,31	62%	984,76 €
RMLG314NVZ	2,29 €	656	260	690	0,13	55%	1.576,74 €
93UUPW7ZW	3,26 €	120	70	107	-0,18	43%	349,41 €
4UK7NA0MMH	2,84 €	0	5	1	0,15	56%	2,29 €
PD5G6RZ2MY	0,59 €	3000	1365	3105	0,08	53%	1.829,20 €
Y8K220ZVC3	0,43 €	1797	737	1816	0,03	51%	772,80 €
FAA22G6T46	1,77 €	60	35	67	0,21	58%	119,10 €
7KJEE3LTX	2,78 €	55	31	70	0,51	69%	195,58 €
6S1524GT5F	5,08 €	144	118	171	0,23	59%	870,77 €
387FEZ314G	4,49 €	176	80	190	0,18	57%	853,24 €
5F34580O8	4,44 €	9	16	11	0,15	56%	50,10 €

Panel B: Summary Results of all 50 Iterations

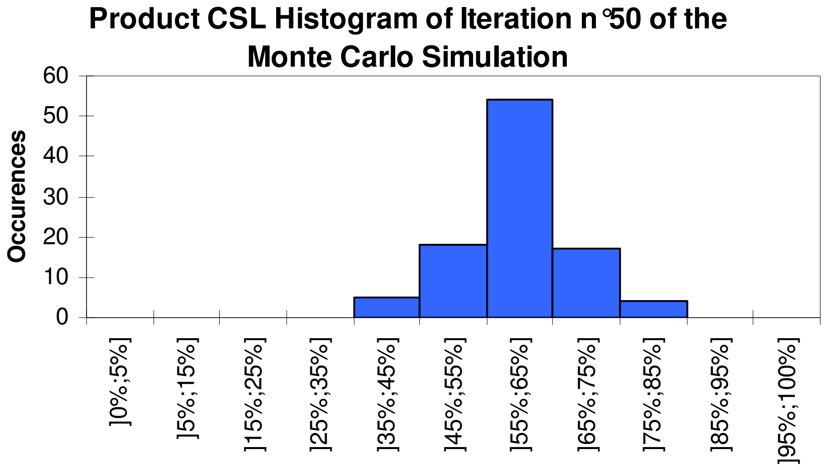
Iteration	Costs	Iteration	Costs
1	119.400 €	26	123.560 €
2	125.863 €	27	122.144 €
3	120.546 €	28	120.968 €
4	123.973 €	29	128.015 €
5	126.772 €	30	121.616 €
6	121.211 €	31	123.562 €
7	127.502 €	32	120.696 €
8	124.007 €	33	125.266 €
9	129.762 €	34	122.357 €
10	123.178 €	35	122.173 €
11	126.298 €	36	128.451 €
12	125.825 €	37	123.202 €
13	123.437 €	38	126.188 €
14	123.266 €	39	124.086 €
15	125.980 €	40	126.779 €
16	128.569 €	41	127.226 €
17	126.934 €	42	122.058 €
18	119.946 €	43	126.939 €
19	121.671 €	44	127.065 €
20	122.966 €	45	127.789 €
21	121.711 €	46	126.639 €
22	126.039 €	47	122.700 €
23	119.713 €	48	124.781 €
24	121.785 €	49	124.917 €
25	125.261 €	50	124.096 €

Minimum:	119,400 €
Maximum:	129,762 €
Average	**124,298 €**

Note:

The Monte Carlo Simulation algorithm starts with a quantity of 0 for each product, and then randomly allocates one unit to any product until enough units have been allocated to obtain an expected customer service of 60% (here defined as the target).

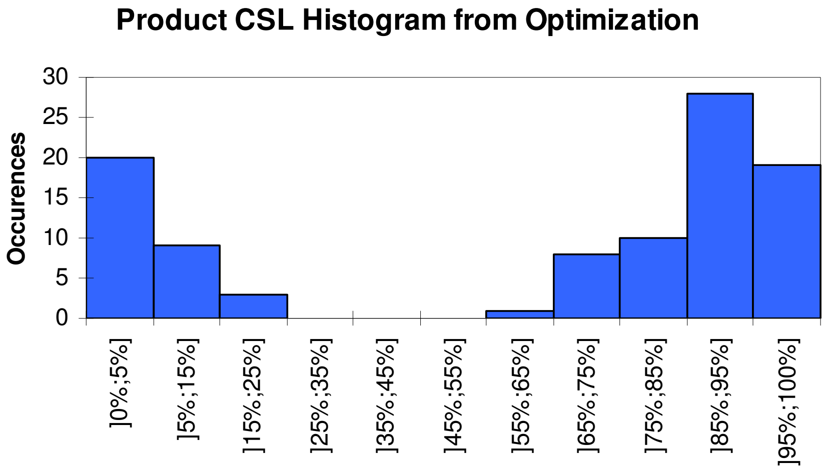
Unlike the optimization algorithm, this algorithm is not concerned with costs. Generally, the CSL for a single product thus varies between 45 and 75%, with only few products falling outside that range (see histogram). In fact, the random allocation produces a normal distribution around the target CSL of 60%.



If all products' CSL was set to 60% in this example, the costs would amount 123,500 €.

This distribution clearly differs from an optimal distribution, in which products with a relatively high security cost (i.e. $SE_i * c_i$ is high) will have a very low CSL and products with a relatively low security cost will have a very high CSL.

As an example, this is the distribution that is obtained using the optimization algorithm with a target CSL of 60% (with exactly the same input data).



The corresponding material investment, for July 2006, would be 39,819 €, i.e. 84,478€ (66%) less than the average cost with a random quantity allocation. This again visualized the extent of the cost saving potential.